THE Arithmeticall JEVVELL:

OR

The vſe of a ſmall Table;

Whereby is ſpeedily wrought, as well all Arithmeticall workes in whole Numbers, as all fractionall operations, without fraction or reduction.

INVENTED

By

William Pratt.

Arts
8.D.75.

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To the High and Mightie PRINCE CHARLES:

Prince of *Wales*, Duke of *Cornwall*, *Yorke*, *Albany*, and *Rothsay*: Marques of *Ormont*, Earle of *Rosse*; and Baron of *Armanoch*: High *Seneschall* of *Scotland*; Lord of the Iles; and Knight of the most Noble Order of the GARTER.

The Preface.

It was likely of late (courteous Reader) to haue loſt mine onely ſonne, on whoſe birth and breed I had beſtowed much care and coſt; but on inquirie, found him at laſt by ſtrangers fire; and fearing that it might in time bee reputed baſe, or at leaſt wiſe marred by ill education, I haue heere taken it home, intreating your kinde teſtimony that I father it as mine owne, with full aſſurance (if not blinded with ouerweening affection) that he will heereafter proue no leſſe pleaſing to me, then profitable to his natiue Country. Yet muſt I needs acknowledge the modeſty

of my Compater or Competitor, to be farre exceeding hirs, who in presence of King Salomon *laid claime to her neighbours childe, for that (as I haue heard by those who haue seene and read his harmelesse workes) he hath not once exprest the Author of this Instrument; whether to bee himselfe, or some other man: And had hee not giuen way to his friend,* I. C. *idlely to iniure mee in his harsh and hobling rimes, I could well haue borne the rest. I must likewise confesse, that I freely acquainted him with all the ordinary vse & operations of this my Table; but of the maine and principall (as to worke fractionall operations in whole numbers) I gaue him no light at all, as may well appeare by his laborious Rules in that kinde. But if by diuulging his booke before me; he hath either disparaged the excellent vse of this my Table, or (which I little feare) preuented my hopes and expectations, and with* Iaakob *bereft* Esau *both of birthright and blessing; The blessing of* Isachar *goe also with him, whereunto I leaue him.*

As

As concerning this my Booke, I haue ſtriued by all poſsible meanes, to reduce the ſame into as ſmall a volume as poſsibly I might, and to the ſcantling of my Table, to the end that both of them being bound together in one (as moſt cõueniently they may) are therby made fit and portable companions of the pocket, on all occaſions. Wherein I haue onely handled (and that as briefely as with conueniency I could) firſt the Notation *as well of ordinary Arithmeticke, as Inſtrumentall computations: then the foure vſuall operations of Arithmetike,* Addition, Subſtraction, Multiplication, *and* Diuiſion; *and laſtly, certaine briefe and compendious* Rules *& examples of the* Golden Rule, *as a taſte only thereof, or an introduction to the ſame.*

In all which, I haue not onely intreated of the ordinary operations in whole numbers; but alſo declared how by this Table, with great ſpeed and incredible eaſe and facilitie, to worke all fractionall operations whatſoeuer, in and by whole numbers,

bers, without vse of fraction or reduction.

For the fruitfull vse and vnderstanding heereof, let me aduise the Practitioner, who desires to benefit himselfe thereby: first to read ouer the whole superficially, and then againe to begin the same, and with more mature consideration (taking the Rules *before him as they lie in order) not onely to read, but with the Instrument respectiuely to practise as hee reades: and in his practise not to fly vnto that, before hee perfectly vnderstandeth this; so shall hee in little time, with great ease attaine to the perfect vnderstanding thereof.*

And notwithstanding (for the former reasons) I haue onely heerein handled these briefe and ordinary rules and operations: Yet I purpose shortly with Gods assistance, by a more large and ample work, (wherein I haue alreadie taken some paines) to make it appeare vnto the world, what strange and wonderfull conclusions, may with infinite ease, and vnex-

pected

pected speed bee wrought by this little Table: not onely in all ordinary operations Arithmeticall, with the extraction of roots, and the rules of practise: but in that great and admirable Arithmeticall Art of Æquation, vulgarly called the Rule of Coss. or Algebra, whereunto this Table is as naturally coapted, as to your Decimall interests, either simple or compound, whereby all troublesome questions of that nature are most easily resolued (in a manner) without worke or operation: wherein I shall dispose of those mine indeuours, as I finde thine approbation and good acceptance of these, which if contrary to mine expectation and desire, I shall yet rest on this resolution, that I haue heere beaten an vnknowne path, and vnclasped a secret (if worthy the name) for a more ingenious age to walke and worke vpon.

To conclude (lest I forme the gates too large for this my little citty) I must truely acknowledge (for auoiding that great sinne of ingratitude) that I am

much

much indeered to my worthy good friend, M^r. Aa. Rathborne, *by whose fauourable assistance, and ingenious helpes, I haue receiued much light and ease in these my labours, which heere I leaue to thy good liking: Desiring the Almightie to prosper these and all other our honest and good indeauours.*

William Pratt.

De Inuentione Gemmæ Arithmeticæ.

PRAT *platted hath a platforme rare,*
To adde, substract, diuide & multiply.
T'extract the roots, Cubike, and Square,
Sans *pen, inke, paper readily.*
And sans *Reduction so it doth surpasse,*
Each Practik *Arithmetik that euer was.*
It workes the Cossiks also very well;
Which Cossiks th'others farre excell.
More vsefull a deuice then Nepairs *bones,*
For what he did at twice, this doth at once.

RICHARD HARRIS.
Theol. Do.

GVIL. PRATTO, *Gemmæ Logisticæ Inuentori* εὐπραττειν.

PRæcellunt homines rationis lumine brutis,
Nec virtus numeros est didicisse minor.
Disquirit ratio, numerus disponit; vt illa
Naturas rerum, sic notat iste locos.
Qui numeris igitur gemmæ super addidit artem,
Ingenuâ in numeros arte meretur opes.
Nam quod pēna nequit, faciat nec calculus, immò
Quod non dena facit virgula gemma facit
Hanc studio inuenit Latitans in carcere *Prattus*,
Sic gemmam è vili stercore Gallus habet.
Pauperies acuit mentes, & sedula promit;
Quod sors ignauo condit opima sinu.

Prodijt ante Author qui Gemmæ prodidit artem;
Prodidit hic artem, perdidit atq; librum.

R. A. D. M.

Aa, Rathbornius in laudem Authoris & singularis inuentionis suæ.

FAire nature nothing from her bosome yeelds,
Which suddenly doth to perfection rise;
As birds and beasts, and plants, and flowers i'th'fields
Yea man himselfe presents vnto our eyes:
And Art (at first the Embryon af mans braine)
doth follow nature as her soueraigne.
Time challengeth this priuiledge alone,
To strengthen the weake infants of our wit;
Which here my'ndustrious friend hath plainely showne,
By witty Rules, and demonstrations fit.
Let any Artist see this skilfull frame,
And he will write (Non vltra) on the same.

To my ingenious Friend, M. *William Pratt.*

VVEre I appointed to set downe a price
Or value of thy gemme at any rate,
I buckster-like the market would so hoise,
That t'were no bargaine for a griping mate,
So doe I lewd and lasie pesents hate:
Yet were this forraine borne or hatcht abroad,
I know the Author would haue greater laud.

But we in England *doe misprise our selues,*
In prising strangers ware before our owne;
We doat herein and proue but idle elues,
Sith theirs to men of iudgement worse is knowne,
Or seldome to such true perfection growne.
Yet we ascribe to Spaniard, French, and Dutch,
More then to native Country-men by much.

But were't in Carcere or in Lusingham,
Or where so'ere thy Iewell *was conceiu'd;*
It skils not much, from thee I know it came,
And from thy witty braine it breath receiu'd,
How ere o'th birth-right thou wast quite deceiu'd,
By such as wanting truth, no manners knew,
Nor ne'r arriu'd where sacred knowledge grew.

Thine THO. BRETNOR. M. M.

In commendation of the workes performed by this excellent IEWELL.

VIew here this patterne, rare for the inuention,
Which seemes but poorely deckt in outward hue:
Yet rich for practise, and easie for direction,
In operation quicke, and certaine true.

Addition readily it doth performe:
Substraction also in another kinde,
Multiplication eake it doth adorne,
Without en trouble, or bearing ought in minde.

It doth diuide in whole number and fraction,
And points the quotient without any trouble,
In numbers great it sets out your Substraction,
Without en error, or operation double.

The Golden Rule, or Rule of Three in fractions
It doth performe with numbers whole and sound,
Sauing much labour, in all seuerall actions,
The like before with pen hath not beene found.

For Nepairs *worthy Rods, of some cal'd bones,*
It setteth downe the numbers by infusion:
As t'were an Instrument fit for the nonce,
To help that excellent and admir'd conclusion.

For what the penne doth doe by helpe of rods,
Is heere infused and sooner wrought by ods:
The Iewell *at once by transmutation then,*
Performes two seuerall workings of the pen.

IOHN IOHNSON

De Gemma Arith.

WEll worthy are thoſe Worthies of great praiſe,
Whoſe learned labours future times may bleſſe,
Euer liuing fame their laſting glories raiſe;
That thoſe vnborne their vertues may expreſſe,
Whoſe conſtant care, and euer ſtudious paine,
Was ſtill to further Countries good and gaine.

Among thoſe beautious branches time brought forth
This famous Iewell, Arithmetikes chiefe Table,
To after Ages like to proue more worth,
Then heart can thinke, or tongue t'expreſſe is able:
Bleſt like the Bay tree, euer let it floriſh,
To the Authors praiſe, and all good Arts to nouriſh.

R. L.

Errata.

PAge 11. line 11 for 11 D. read 11. p. 11 3. 15. for figure 8. r. ſigne 8. p. 25. l. 4. for and the r. and adde the. p. 26. l. 6 for 6 D. r. 68 p. 31. l. 25. for and the r. and adde the. p. 32. l. 8. for propertie r. properties. p. 34. l. 17. for the 7. r. that 7. p. 37. l. 21. for in this 1. *read* in the. 1. p. 41. l. 14. for or lower r. or third lower. p. 43 l. 10. for ordinary of r. ordinary courſe of. p. 47. l. 2. for any Number r. any whole Number. p. 49. l. laſt for tiplicand r. Multiplicand. p. 67. l. 5. for produſts 8. r. produſts of 8. p. 72. l. 3. for which with r. which 4 with.

THE Notation, Definition, *and Discription of* ARITHMETIKE: and the *Arithmeticall Jewell.*

ARrithmeticke is *The Art of Computation or Numbring*: by meanes whereof all reckonings and accounts in humane affaires are truly and exactly made and rectified.

The matter, ground, and subiect of this Arithmeticall Art, is *Number*, being *A Mathematicall discreet quantity*, or a multitude of Integers or vnites, agregated and collected together into one summe.

An Integer or vnite is, *any one entire and whole thing of whatsoeuer kinde*, as

one man, one day, one yard, one pound, &c. Many of which being collected together, and growing by their pluralitie to a separate and discreet quantity make a *Number*.

From vnitie proceedeth two boundlesse issues of contrary kinde, *Number*, and *Parts*, that infinitely increasing, and this infinitely decreasing in valuable quantity.

Whereby it commeth to passe, that the greatest whole number, and the least fractionall part, can by no meanes bee expressed or imagined. And of one or both of these kindes doe all Arithmeticall operations consist.

The figures or numerall Characters, whereby all numbers bee they neuer so great or many, may be expressed; are in number ten, being these which follow, 1. 2. 3. 4. 5. 6. 7. 8. 9. 0. whereof the first towards the left hand signifieth one, the second two, the third three, and so forward of the rest in order til you come to the tenth and last, being called a Cypher, which of it selfe signifieth nothing, where, or howsoeuer it be placed, yet filleth vp a roome, and thereby augmenteth the signification in any number, of

all

all other figures placed aboue it towards the left hand.

But it is to bee vnderſtood that notwithſtanding euery of theſe Characters or figures, taken ſolely and ſeuerally by himſelfe, ſignifieth but his owne proper value as before, yet if any number of them bee taken and ioyned together in one ſumme, then haue they another valuation in reſpect of their ſituation, and the roome or place wherein they ſtand, as hereafter appeareth.

The order of the roomes and places in any number, is alwayes to bee reckoned and accounted from the right hand towardes the left, and as many figures or characters as any number hath, ſo many roomes, places, or termes is that number ſaid to containe; as this number, 6732 is ſaid to containe foure roomes, places, or termes, becauſe it hath ſo many figures, whereof the laſt and loweſt figure towards the right hand, being that wherein the 2. ſtandeth, is called the firſt place, the next towardes the left hand being 3. is called the ſecond place, the next that way, the third; and the next & laſt where 6. ſtandeth, the fourth place; and ſo forward with the reſt, if more were.

Which being vnderſtood, it is next to bee conſidered that the ſignifycation of each figure, in reſpect of the roome or place wherein it ſtandeth, is thus: euery figure in the firſt place ſignifieth onely his owne proper value, as 2. in the former number, becauſe it ſtandeth in the firſt place ſignifieth onely two, being his owne proper value; but any figure ſtanding in the ſecond place, ſignifieth tenne times his owne value: and therefore the figure of 3. ſtanding in the ſecond place of the former number, ſignifieth thirtie, which is tenne times 3: wherefore thoſe two figures in the firſt and ſecond places, taken by themſelues ſtanding thus; 32. ſignifie in value together thirty two. Euery figure ſtanding in the third place, ſignifieth an hundred times his owne value, and therefore 7. ſtanding in the third place of the former number, ſignifieth ſeuen hundred: ſo that thoſe three figures ſtanding together thus, 732, are thus to be read and valued, Seuen hundred thirty two. Euery figure in the fourth place ſignifieth a thouſand times his owne value, & therefore 6. in the former number, becauſe it ſtandeth in the fourth place, ſignifieth 6. thouſand. Wherein is to be noted

that

that euery figure in the firſt place, is ſo many vnites; in the ſecond ſo many tens; in the third, ſo many hundreds; in the fourth ſo many thouſands; as the figure in it ſelfe containeth vnites: as 1. in the firſt place is one vnite, in the ſecond one tenne, in the third one hundred, in the fourth place one thouſand: ſo that foure of them being placed together in one ſumme thus 1111; are thus to bee read, one thouſand one hundred and eleuen. In which number notwithſtanding euery figure is one and the ſame, yet by reaſon of the difference of their place and ſituation, each figure exceedeth other in a due proportion ten fold; as the figure 1. in the firſt place is onely one, the ſame figure 1. in the ſecond place is ten times as much as that in the firſt, namely ten: the ſame figure 1. in the third place is tenne times as much as that in the ſecond, namely one hundred: for 10 times 10 is 100: and the ſame figure 1. in the fourth place is ten times as much as that in the third, namely one thouſand for tenne times one hundred makes one thouſand, and ſo forth, infinitely, each vnite exceeding other in reſpect of his next higher roome tenne fold. Which is the principall reaſon and

theoricall ground as well of mine *Arithmeticall Iewell* : as of many other excellent conclusions.

By which ten fold increase, iudicially noted and well vnderstood, the meanes is apparant & very easie at the first view, as well to read any number giuen, as to expresse any number required, yet for your more ease and helpe herein, it were not amisse to vse this following rule.

If a great summe of many places or figures bee giuen to bee read, as 367340217432 00 : consisting of fifteene figures or places : First directly ouer the fourth figure or place from the right hand, which in the former number is 2. make a pricke or point, and the like directly ouer euery third figure or place from thence towardes the left hand ; so shall you haue in this number foure prickes or points, and betweene each point two figures, as appeareth in the example : Now supposing that by those points the whole number is diuided into fiue terneries or seuerall numbers, of three figures in each, by the rule formerly taught, you are to read the first ternery towardes the left hand thus : three hundred sixty seuen, the second ternary following

lowing, fiue hundred and forty, the third ternery 2. hundred & fifteene, the fourth ternery, seuen hundred forty three, and the fift and last, two hundred. The difference betwixt which, and reading of it as one whole and intire number, is no otherwise then thus, at the end of the reading or pronouncing the first ternery towards the right hand, giue an addition of so many thousands as there are points ouer the whole number, which in the example is foure, at the end of the second ternery giue an additiō of three thousand, for the three points then remaining: at the end of the third ternery, two thousands; and at the end of the fourth, one thousand. So shall the giuen number bee thus pronounced in reading, three hundred sixty seuen thousand, thousand, thousand, thousand, fiue hundred and forty thousand, thousand, thousand, two hundred and fifteene thousand, thousand, seuen hundred forty three thousand, and two hundred; And the like of all others bee the figures or places therein neuer so great or many.

And here from this last giuen number ariseth another doctrine worthy of note, shewing the nature, passion, and proper-

tie of a Cypher, for as before I partly declared, that notwithſtanding a Cypher of it ſelfe is of no worth or value, yet hath it the nature and propertie of many excellent Artiſts, who though ſimple, poor, and meane in themſelues, yet mainely multiply and increaſe others: as let the laſt cypher in the former number bee taken away, and his whole company of figures in the ſame number ſhall bee tenne times meaner in value then now they are, the firſt ſignifying figure towardes the right hand being now two hundred, is then but twenty, the next being now three thouſand, is then but three hundred, and the like of the reſt: But take away both the laſt cyphers, then ſhall all the reſt be an hundred times meaner then now they are, as the firſt figure towards the right hand now two hundred, then ſimply 2. the next figure now three thouſand, then only thirty, and ſo of the reſt. And the like conſideration is to bee had betweene the cypher towards the middle of the former number, and thoſe fiue ſignifying figures aboue the ſame, for to ſuch as are after or vnder him in degree or place, hee hath no reſpect neither by increaſing or decreaſing their worth or value.

Inte-

Integers or whole numbers consist of three seuerall sorts or kindes, That is, *Digits*, *Articles*, and *Mixt Numbers*.

A *Digit number* is that which may bee written and expressed onely with one figure, as 1. 2. 3. 4. &c. of which kinde there is onely ten, whereof the first nine are signifying, and the tenth is a cypher.

Article Numbers are such as haue alwaies in their first place a cypher, as 10. 20. 30. 100. 110. 120. and the like.

Mixt Numbers are all such as are aboue ten, and not hauing any cypher in the first place thereof, for notwithstanding they haue cyphers in any other place, yet are they not called *Articles*, but *Mixt Numbers*, becausse they are compounded and mixt of *Digits* and *Articles*, as 11. 12. 17. 123. with others *ad infinitum*.

And thus much briefly concerning the Notation and definition of whole numbers, according to vsuall operation; and heere might I likewise aptly insert, the discription and notation of fractions and fractions of fractions, with their diuersitie of kindes, which for two speciall reasons I will omit. First for that in this small volume I wil striue to be as briefe as possibly I

I may, and ſecondly and moſt eſpecially, for that I purpoſe not at all in the vſe of my Inſtrument, to peſter my ſelfe or my practitioner with thoſe troubleſome rules of fractions, but wholly to worke in whole numbers all fractionall opperations, whatſoeuer can be propoſed, without fraction or reduction; wherefore before I enter into the materiall parts of Arithmeticke, I will here briefly touch the definition, deſcription, and notation, as wel of this kinde of operation, as of mine Inſtrument, or Arithmeticall Iewell.

Firſt, in the head or vpper margent of this Table, is expreſſed from the right hand towards the left, the ſeuerall ſignes of each roome or place ſignifying their increaſe in value and eſtimation, the three loweſt towardes the right hand ſeruing for ſhillings, pence, halfe pence, and farthings; the fourth place being that next vnto ſhillings is the place of vnites, the next of tens, then hundreds, thouſands, &c. and ſo each place exceeding other by a ten fold increaſe, to the laſt and higheſt towards the left hand.

Directly vnderneath which ſeuerall ſignes, there are ſeuerall orders of characters, or figures thus diſpoſed In the firſt

and

and loweſt place towards the right hand, vnder this little (q;) are placed three prickes or points ſignifying ſo many farthings, (as needleſſe of more) for that a fourth maketh one penny, which is a vnite in the next higher roome towardes the left hand: In the ſecond roome vnder this ſigne (d) for pence, are placed theſe two letters (v i) ſignifying ſix, and alſo 5. prickes or points, which together with the former ſix make 11 d. for ſo many pence (as needleſſe of more) for that one more maketh a ſhilling, which is expreſſed by a vnite in the next higher room: In the 3. room vnder this figure s. for ſhillings are placed theſe two letters xv. ſignifying fifteen, and alſo 4. prickes or points, which together with the former fifteene make 10. for ſo many ſhillings (as needleſſe of more) for that one more makes xx s. or one pound, which is expreſſed by an vnite in the next higher roome being the place of vnites, or firſt of integers, and in euery other roome and place towardes the left hand, throughout the whole Table, is placed the letter v. with 4. prickes or points, ſignifying together 9. vnites (as needles of more) for that ten vnites in any one

placce

place is equall in value to one vnite in the next higher roome, one vnite in any one place being the tenth part of one vnite in the next higher roome. Wherefore if you are to expresse ten in any one roome, (as in the third place, or place of hundreds) then are you to open one vnite in the next higher roome, being the place of thousands, for ten hundred make one thousand.

If you are to expresse 20. 30. or 40. in any place, then open 2. 3. or 4. vnites in the next higher roome, by the former reason.

And hereby it is also manifest, that the letter v. or 5. in any one place, is halfe of an vnite in the next higher room, for 2. times 5. makes 10. Againe, if v. bee $\frac{1}{2}$ of an vnite in the next higher roome, then $\frac{1}{4}$ thereof is as easily had, the halfe of 5. being $2\frac{1}{2}$ which is expressed by opening of two vnites in the same place vnder the v. and by opening of the v. in the next lower roome: And heere is also had $\frac{1}{4}$ for $\frac{1}{2}$ and $\frac{1}{4}$ makes $\frac{1}{4}$; and thus are these fractions easily expressed in whole numbers, and fit for any operation: But to shew the full and compleat order and manner of expressing of all fractions to bee wrought

wrought in whole numbers, being vnfit for this place, I will heere forbeare till I come in due order to those rules.

The precedent directions well vnderstood, I hold it most plaine and easie for the meanest capacity, in this plaine and ordinary manner of computation, as wel speedily to expresse any number required, as readily to read any number deliuered. As let it be required to set downe or expresse in the Table this number; 5603482. First consider how many places or figures are contained therein, which are 7. and therefore the 5. being the highest figure towards the left hand, is 5. millions; wherefore place the same in the seuenth roome in the Table being the place of millions, and all the rest in order after it, as heere they stand in the giuen number.

Againe let it bee required to expresse in the Table, 467.l.-13.s.-6½.d. First open 4. in the place of hundreds, then 6. in the place of tens, and 7. in the place of vnites, 13. in the place of shillings, 6. in the place of pence, and 2. in the place of farthings. And as for reading these, or any other numbers required after this manner of ordinary computation, consi-

dering

dering that the ſeueral ſignes & denominations of each place is particularly expreſſed ouer the ſame as before, nothing can bee more plaine and eaſie, and therefore will I ſpend no longer time herein.

The Theoricall ground and reaſon of this Inſtrumentall Arithmeticke (for ſo I ſtile it) dependeth (as I haue partly touched before) on the tenth progreſſion geometricall; ſo that whatſoeuer number bee propoſed and placed in my Table, it is to bee vnderſtood that the vnite of euery particular terme, therein exceedeth the vnite of each terme in the next lower roome towardes the right hand ten fold; as let 324. bee a number giuen and expreſſed in the Table, then I ſay that euery vnite in the terme or figure 3. exceedeth each vnite in the terme or figure 2. in the next lower roome tenne fold; for each vnite in the figure 3. is 100 and each vnite in the figure 2. is but 10. and the like betweene the vnites of 2 and 4. in the ſame number: and ſo of all others.

And ſeeing this rule holds generall in this kinde of Arithmeticke, as well in whole numbers as fractionall termes, and that we haue vndertaken in whole numbers,

bers to performe all fractionall operations, I hold it a maine & principall point, Firſt to finde out a ſufficient meanes to diſtinguiſh in all operations thoſe numbers, which ſignifie intigers, from thoſe which ſignifie fractions, conſidering there is no difference at all in there deſcription or notation: As let this number be giuen and thus expreſſed in the Table, 245. whereof let the 2. firſt figures towardes the left hand being 24. bee Integers, and the laſt 5. towardes the right hand, a fractionall terme, which together import 24 ½ and in knowne parts of coine 24 l-10 s-0 d. for that the 5 is ½ of an vnite in the next higher roome, as before was declared; but here is no apparant or actuall diſtinction, wherefore euer in my following rules, as in all opperations with the pen, I wil alwaies diſtinguiſh the integers from fractions thus, 245, placing ouer each fractionall terme a pricke or point, and leauing the integers without. As here likewiſe in this number 8493625 which after the vſuall manner is thus expreſſed, 8493 ½ and in knowne parts of coine thus 8493 l-12 s-6 d, as ſhall hereafter appeare.

But ſeeing theſe prickes or points ſerue onely

only for opperations by the pen, & to no vſe or purpoſe with the Table, in ſtead thereof I alwayes make choice in my Table of any one parallel line fitteſt for my purpoſe, according to the nature of my preſent work, which I cal the Prime line, or line of Diſtinction, as well for that it is next aboue the firſt fractionall terme, which is called a prime, as that it ſerueth to diſtinguiſh the integers from thoſe fractionall termes, at the foot of which line I alwaies open an vnite, to make it more apparent to preſent view. And then I euer place my Integers in order aboue that line towardes the left hand, and my fractionall termes beneath the ſame towardes the right hand thus 849 | 625.

In all inſtrumentall numbers, whatſoeuer giuen or required, I call the whole numbers integers or vnites, and the ſeuerall fractionall termes according to their ſituation and place, as let this number bee giuen, 8957625. Now the 895. I call integers or vnites, and the fractionall termes in order as they ſtand, *primes, ſeconds, thirds*, and *fourth, viz.* the firſt 7. primes, the next 6. ſeconds, the next 2. thirds, and the laſt 5 fourths, and ſo forth

forth infinitly in order as many as are giuen or required : The ſeuerall values of which 4. fractional termes are thus particularly $\frac{7}{10}$ $\frac{6}{100}$ $\frac{2}{1000}$ $\frac{5}{10000}$ and together thus, $\frac{7625}{10000}$ taking alwayes the fractional termes for Numerator & as many cyphers with an vnite before them, for Denominator; which fraction may bee ſpeedily reduced into his ſmalleſt termes (as ſhall be hereafter ſhewed) and being ſo reduced importeth $\frac{61}{80}$.

It is alſo to bee vnderſtood that in all theſe inſtrumentall numbers, the tenth part of each vnite in the laſt and loweſt integer towardes the right hand makes a *prime*, the tenth part of the vnitie of a prime makes a *ſecond*; the tenth part of the vnity of a ſecond, makes a *third*, &c. infinitely: whereon it followeth, that the greateſt fractionall terme in it ſelfe neuer exceedeth 9. wherefore wee cannot ſay, 5. *primes*, 12. *ſeconds*, but in ſtead thereof 6. *primes*, 2. *ſeconds*, nor 3. *ſeconds*, 15. *thirds*, but in ſtead thereof 4. *ſeconds*, and 5. *thirds*, which are of like value.

And conſidering that oftentimes (in this worke with the pen) occaſion ſhall be offered to expreſſe fractionall termes alone, without integers, wherein I ſhall not

not alwayes beginne with primes, but ſometimes with *ſeconds*, *thirds*, &c. and if in ſuch caſe I ſhould onely vſe prickes or points ouer them, it ſhall ſerue only to declare that they are fractionall termes, but not of what nature; wherefore alwaies if my firſt termes towards the left hand bee primes, in ſtead of a point, I place ouer it the figure 1. and points ouer the reſt; If *ſeconds*, the figure 2. & points ouer the reſt; If *thirds*, the figure 3. and the like of all others. For it is to bee vnderſtood, that notwithſtanding for breuitie and conuenience, I vſe points or prickes, yet the proper ſigne of a *prime* is 1. of a *ſecond* 2. of a *third* 3. &c.

Alſo it is to be conſidered that oftentimes, and in moſt opperations theſe Inſtrumentall numbers are to be taken and vſed for ſums of money in coine: wherefore I hold it fitting briefely to expreſſe what theſe fractionall termes, of *primes*, *ſeconds*, and *thirds*, &c. doe import in the knowne parts of coine.

Firſt therefore let it bee vnderſtood, that in the place of *primes*, euery vnite is in value of coine 2 s. for as the vnite of any place is the tenth of a vnite in the next higher roome: ſo is 2 s. being an vnite

vnite in the place of primes, the tenth of one pound which is an vnite in the next higher roome; wherefore the double of any number found in the place of primes, is so many shillings as 2 there, is 4 s. 3. is 6 s. 4 is 8 s. & 5 (which is alwayes the halfe of an vnite in the next higher roome) is 10 s. and the like of the rest.

	l.	s.			
So then	3	16	is thus expressed.	3	and the like of others.
	24	8		24	
	36	10		36	

In the place of *seconds*, euery vnite is in value of coine, two pence, and two fifts of a penny, euery two vnites there, is $4\frac{4}{5}$d. 2 vnites $7\frac{1}{5}$s. 4 vnites $9\frac{3}{5}$; and therefore the 5 in the same place is 12 d.

	l.	s.	d.			
So then	5	1	$3-2\frac{2}{5}$	is thus expresst.	5	and the like of others.
	46	1	$5-4\frac{4}{5}$		46	
	324	19	$-7\frac{1}{5}$		3249	

In the place of *thirds* euery vnite is in value coine $\frac{1}{5}$ of $1\frac{1}{5}$d. two vnites is $\frac{2}{5}$; three vnites is $\frac{3}{5}$; and foure vnites is $\frac{4}{5}$ of $1\frac{1}{5}$d. wherefore the 5. in the place of *thirds* (being $\frac{1}{5}$ of an vnite in the place of *seconds*) is $1\frac{1}{5}$d.

So

	l.	s.	d.			
So then	15	6	6	is thus expreſt.	163	and the like of others.
	23	4	3		232125	
	89	10	9		895375	

As concerning all other inferiour degrees of fractionall termes, as *fourths*, *fifths*, &c. being of very meane value; (an vnite in the place of *fourths*, being little more then ¼ mite, and in *fifths*, little more then ⅕ mite) the meanes of obtaining their preciſe quantities, together with the reduction of any fraction whatſoeuer, giuen or required into theſe Inſtrumentall termes (whereby they may be ſpeedily wrought in whole nombers) ſhall bee hereafter taught when we come to operation, being vnfit for this place.

And hauing thus briefly handled the Definition, Deſcription, and Notation I wil next proceed with the material parts of Arithmetike, as the foure operations Arithmeticall, *Addition*, *Subſtraction*, *Multiplication*, and *Diuiſion*; of which in order as followeth.

Of Ad-

Of Addition; the first Arithmeticall Operation.

Definition.

ADDITION, is the vniting or gathering together of many particular parcels, into one totall ſumme, as 4. 6. & 8, being three particular parcels, are by this operation vnited and gathered together into one totall ſumme, making 18.

RVLE I.

To adde whole Numbers or Integers together into one totall ſumme.

LEt theſe ſeuerall numbers be giuen, and let it bee required to adde them together into one totall ſumme. Firſt, collect thoſe termes together which are in the higheſt places towardes the left hand (which is contrary to operation with the penne) ſaying thus: 8 and 4 is 12, and 6, is

63897
47082
80564
3758
4600
7534

is 18. And considering they ſtand in the fift place, or place of tens of thouſands, therefore in the fift place of the higheſt ranke in the Table place the Digit 8, and the 1 in the next higher roome towards the left hand, being the fixt place, thus, 18; then collect together the next order of termes, ſtanding in the place of thouſands, ſaying 7 and 4 is 11, and 2, is 14, and 7, is 21, and 3 makes 24, whereof place the Digit 4, in the next lower roome to the 18, in the Table formerly placed, and adde the figure 2, to the 8 there already placed, which together make 10, and ſeeing that 10; in any one roome is an vnite in the next higher roome, therefore cancell the 8, and adde an vnite in the next higher roome, to the vnite there already placed. Then collect together the next lower order of termes, being in the place of hundreds, ſaying thus; 5 and 6 is a 11, and 7 is 18, and 5 is 23, and 8 makes 31, whereof place the Digit 1, in the next lower roome in the Table, to the 4 there laſt placed, and adde the 3 to the ſame 4, which together make 7. Next collect together the next lower order of termes, being in the place of tens, ſaying thus, 3

and

and 5. is 8. and 6. is 14. and 8, is 22. and 9. is 31. whereof place the Digit 1. in the next lower roome, in the Table to the 1. there laſt placed, and adde the 3. to the ſame 1. ſo there before expreſſed, which together make 4. Then laſtly collect together the laſt and loweſt order of termes, being in the place of vnites, ſaying thus; 4 and 8. is 12. and 4. is 16. and 2. is 18 and 7. makes 25. whereof place the Digit 5. in the next lower roome to the 1. there laſt placed, and adde the 3. to the ſame 1. making together 4. ſo ſhall your worke ſtand thus, 207435. which is the totall of the ſeuerall numbers giuen to be added together as was required.

It is to bee noted, that notwithſtanding the numbers here giuen to be added together, are in all but 6. yet by this rule and Inſtrument may bee added together, as many numbers as can be giuen or required; for hauing firſt collected together as many as with conueniency you may, and making ſome ſmal priuat note or marke with your pen or bodkin wherewith you worke, how farre you haue collected, you may there againe begin, and ſo proceed infinitely, as before is taught, adding the ſeuerall particulars together, till you haue the generall

rall totall ; which worke is very ſpeedily wrought and much better vnderſtood by ſmall practiſe, then many wordes; which inſtruction is generall, as well for this as the following Rules of this firſt Operation.

Rvle II.

Diuers Numbers being giuen in knowne parts of coine, to adde them together into one totall ſumme.

SVppoſe theſe 5 ſeuerall ſummes of money were owing, and let it be required to know the totall. Firſt, as before is taught, beginne with the higheſt order of Integers towardes the left hand, ſaying thus, 9 and 8 is 17. and 6. is 23. and 4 is 27; and conſidering this order of termes, is in the place of hundreds, place your Digit 7. in the third roome of your higher ranke in the Table, being the place of hundreds, and the 2. in the next higher roome, being the place of thouſands, where they ſtand thus, 27. then collect together the next lower order of termes, being in the place

l.	s.	d.
436	13	6
625	14	8½
857	2	7¼
979	5	6
96	12	8

place of tens, saying 9. and 7. is 16. and 5. is 21. and 2. is 23. and 3. makes 26: whereof place the Digit 6. in the next lower roome after the 7. and the 2. to the same 7. which make together 9, so shall your worke stand thus, 296. then collect together the next lower order of termes, being the last of Integers, standing in the place of vnites, saying, 6 and 9. is 15. and 7. is 22. and 5. is 27. and 6. makes 33. whereof place the Digit 3. in the next lower roome after the 6. already placed, and adde the other 3. to the same 6. which together make 9. so shall your workes stand thus, 2993. Then next proceed to the place of shillings, collecting together first the *tens*, by saying 1. 1. and 1. makes 3. which is 3. times ten shillings, that is 30 s. or 1 l–10 s. for which open the x. in the place of shillings, and an vnite in the next higher roome (being the place of vnites) for 1 l. so shall now your worke stand thus, 2994 l–10 s. then next collect together the Digits in the place of s. saying 2. and 5. is 7. and 2. is 9. and 4. is 13. and 3. is 16; being so many shillings, which added to the x s. formerly placed in the Table, makes 26 s. wherefore cancell that

former x s. and in ſtead thereof place 6. and open an vnite more in the place of vnites, for the 1 l. by cancelling the 4. there formerly placed, and opening the 5. in the ſame roome, ſo ſhal your worke now ſtand thus, 2995 l--6 d. then next collect your pence, ſaying, 8. & 6. is 1 s. 2 d. and 7. is 1 s-9 d. and 8. is 2 s-5 d. & 6. is 2 s-11 d. whereof place the 11. in the place of pence, and the 2. in the place of ſhillings, ſo you haue now 2995 l-8 s. 11 d. Then laſtly, collect together your halfe-pence and farthings which together make 3 q; which place in the place of farthings, ſo ſhall your whole collection be 2995 l-8 s-11 d ¾ d. being the totall of the ſeuerall ſummes giuen to be added together, as was required.

Of Ad-

RVLE. III.

To adde together any number of Instrumentall numbers giuen or required, as Integers, Primes, Seconds, &c. Whereby all fractionall additions whatsoeuer may bee wrought in whole numbers, without vse of fraction or reduction.

SVppose these 5. seuerall numbers were giuen, which may import either as many seuerall summes of money, or so many seuerall quantities of any other denomination (as shall hereafter appeare) and let it be required to finde their totall without vse of fraction or reduction. In all operations of this nature, you are first in the Table, among the parallel lines, there to expresse and note out (as before is spoken) your *Prime* line, or line of distinction, (whose office is to distinguish the Integers from the fractionall termes) which is thus to be performed.

3624	346875
6345	840625
972	009375
87	9125
9	4375

Note amongst all your giuen numbers, what is the greatest fractionall terme, or more properly, what is the greatest signe

of your fractionall termes, which in this example is 6. (for the first 3. of these giuen numbers, haue euery of them 6. fractionall termes) wherefore take in the Table the sixth parallel line, from the place of shillings for your *Prime* line, which note or marke out by opening an vnite at the foot thereof, to make it apparant at the first view, so haue you 6. places towardes the right hand of your *Prime* line, wherein to place the fractionall termes of your totall (for more then are an the giuen termes, there can neuer be in the totall, but often lesse) and of the Integers, the termes of their totall can neuer be lesse, but most vsually greater or more in number then the greatest of the giuen Numbers. Now hauing thus found out and expressed your *prime* line; you are next likewise to consider, what is the greatest number or highest place of Integers, amongst all your giuen numbers, which at first sight appeareth to bee 4; wherefore in the fourth place from the *prime* line towardes the left hand, is the first terme or figure of your totall to stand and be placed, being now to bee reckoned the place of thousands; for in these operations the place of vnites

or first place of Integers is euer to be reputed that which is next vnto the *prime* line towards the left hand; which being well vnderstood and remembred, there is now no other ambiguity, nor any difference at all in operation betweene this & that of whole numbers in the former first *Rule*: yet for your better ease & practise, I will proceed herein thus. Begin (as in the 1. *Rule*) with collecting those termes which are in the highest place towards the left hand of the giuen numbers, saying 6. and 3. is 9: which place in the fourth place aboue the *prime* line, towardes the left hand: then collect together the next lower order of termes, saying, 9. and 9. is 12. and 6. is 18; whereof place the Digit 8. in the next lower roome vnto the 9. there formerly placed, and adde the 1. vnto the same 9. which together makes 10; wherefore cancell the 9. and place a vnite for 10. in the next higher roome, so shall your worke stand thus, 108. Then next collect the next lower order of termes, saying, 8. and 7. is 15. and 4. is 19. and 2. is 21, whereof place the Digit 1. in the next lower roome, to the 8. there last placed, and adde the 2. to the same 8. which together make 10.

wherefore cancell the ſame 8 and place a vnite for 10. in the next higher roome; ſo ſhall your work ſtand thus, 1101. then collect the next lower order of termes, as 9. 7. 2. 5. and 4. which together make 27. whereof place the Digit 7. in the next lower roome, being heere the place of vnites, and adde the 2. vnto the 1. in the next higher roome, and your worke ſhall ſtand thus, 11037. & thus are your Integers finiſhed, but without difference. Proceed in like order with the reſt, collecting together the *primes* (being the next lower termes) that is, 4 and 9. 8. and 3. making together 24, whereof place the digit 4. in the next lower roome, and the 2. adde vnto the 7. in the next higher roome, which together make 9, ſo ſhall your worke ſtand thus, 110394. Then collect together thoſe termes in the place of *ſeconds*, as 3. 1. 4 and 4, which together make 12, whereof place the digit 2 in the next lower room, vnto the 4. there already placed, which is the place of ſeconds, and adde the 1. vnto the ſame 4. in the place of *primes*, cancelling the 4, there & opening the 5. in the ſame room, ſo ſhall your worke ſtand thus, 1103952. Then collect together theſe termes in the

place

place of *thirds*, as 7. 2. 9 and 6, making together 24, whereof place the digit 4. in the next lower roome, being the place of *thirds*, and adde the 2 vnto the 2. in the place of *seconds*, which make together 4. and so shall your worke stand thus, 11039544. Next collect together those termes in the place of *fourths*, as 5. 5. 3. 6 and 8, which make 27, whereof place the 7. in the next lower roome, being the place of fourths, and adde the 2. to the 4. in the place of *thirds*, which together makes 6. *thirds*, and your worke shal stand thus, 11039547. Then proceed to those termes in the place of *fifts*, collecting them together thus, 7. 2 and 7. makes 16, whereof place the 6. in the place of *fifts*, and adde the 1. to the 7. in the place of *fourths*, which together makes 8. *fourths*, and thus stands your worke 11039546. And lastly, collect the last order of termes, as 5. 5 and 5, which together make 15, whereof place the 5. in the next lower roome, being the place of *sixths*, and the 1. to the 6. in the place of *fifts*, which together make 7, and so shall your last worke stand thus; 110395467, being the totall of the seuerall Instrumentall Numbers giuen,

which valeweth according to ordinary operation with fractions 11039$\frac{3}{4}$, or being esteemed as knowne partes of coine, 11039 l–10–2–11$\frac{1}{4}$ d. which operation is wrought wholly in whole Numbers, without Fraction, or Reduction, as was required.

The property belonging properly to Addition.

If equall numbers be added to equall, their totals shall be equall: but if equall numbers be added to vnequall, their totall shall be vnequal; as if 6 and 6. being equall numbers, bee added to 3 and 3, which are likewise equall; then shal their totalls bee 9 and 9. which are also equall together; but if 6 and 6, being equall shall be added to 3 and 7, which are vnequals, then shall their totals be 9 & 13. which are also vnequals, *Euc.* 1. *Com. Sen.* 2. 5.

The proofe of Addition.

The best and surest proofe of Addition, is by Substraction, being the next following operation, and therefore the proofe of this shall be taught in the conclusion of that.

Of

Of Substraction, the second Operation Arithmeticall.

Definition.

SVBSTRACTION, is an abatement or deduction of one number from another, being either greater, or equall vnto it, whereof the greater number is called the *Grose*, or *totall*, and the lesse, the *Abatement*, whereby alwaies is produced and found a third number, which is called the difference or remainder.

RVLE. I.

Two vnequall whole Numbers being giuen; to substract the one from the other.

SVppose that *A.* being an accomptant to *B.* for the summe of 896547 l. produceth his accounts, whereby it appeareth that at seuerall payments he hath made satisfaction to *B.* of 787681. of the former charge. And let it bee requi-

red to know what is ſtill remaining due on this account. Firſt place in the Table the *Groſſe*, or *totall* in the higheſt ranke, euery terme in his due place, according to the ſignes expreſſed in the head of the Table to that purpoſe, and the *Abatement* directly vnderneath in the ſecond ranke, ſo ſhall your number ſtand thus; 8965472 7876385 then begin your operation with the firſt figures toward the left hand (being contrarie to the worke of the pen) ſaying, 7 out of 8. reſts 1, wherefore cancell or eclipſe 7 of the 8. and leaue 1. remaining in the ſame roome where 8. now ſtandeth, then deduct 8. the next lower terme out of 9. directly ouer it, ſaying, 8 out of 9. reſteth 1. and cancell 8 of that 9 and leaue 1. in the ſame roome; then conſidering the 7. the next lower terme cannot bee deducted out of 6. the terme ouer it, therefore ſay, 7 out of 10 reſteth 3. and cancell the laſt 1. in the next higher roome, which is 10. and adde the 3. which was remaining vnto the 6. ſtanding ouer the 7. which together make 9. Likewiſe conſidering that the 6. in the next lower roome will not be abated out of 5. ſtanding ouer it, therefore ſay, 6. out of 10. reſteth

resteth 4. for which 10, cancell an vnite in the next higher roome vnto the 5, making the 9. there 8. and adde the remainder 4. vnto the 5, which together make 9. Then next abate 3. the next lower terme out of 4. standing ouer it, saying, 3 out of 4 resteth 1, wherefore cancell 3 of that 4, and leaue 1. in the same roome; and lastly, for that 8. the last terme cannot be abated out of 7. ouer it, say 8. out of 10 resteth 2. and for the 10. cancell the vnite in the next higher roome, vnto 7. and adde the remainder 2. vnto the same 7. which together make 9.

So shall your worke stand thus;

108909 the first or vppermost of which
787638 two Numbers is the *difference* or *remainder* required: the truth of which operation is instantly proued, by adding those 2. numbers together as was taught by the 1. *Rule* of Addition, whereby if you produce in stead of this *Remainder* the *Grosse* or *totall* giuen, that is, 896547 then haue you truely wrought, or otherwise not.

RVLE. 2.

Two vnequall Numbers being giuen, (the first terme of the greater being an vnite, and all the rest cyphers) to substract the lesse from the same at the first view, without any operation at all.

LEt these two numbers be giuen, and let it bee required at the first view, without operation to expresse the difference or remainder (the lesse being substracted from the greater.) The two giuen Numbers being placed in the Table in due order, as heere they stand, note well the seuerall termes in the abatement or lesse number, and take their seuerall complements or supplements vnto 9, and likewise the complement of 6 s. 8 d. to 1 l. for the difference or remainder required: So then, I affirm at the first view of the giuen Numbers that 12345367 l. 13 s. 4 d. is the difference or remainder required, as plainely appeareth this remainder being thus placed vnder the abatement, as here they stand; for euery terme

	l.	s.	d.
	10000000		
	87654632	-6-	8
	12345367	-13-	4

terme in the lower number, being added to each terme ouer it, makes iustly 9. and the 13 s-4-d. in the lower number being added to the 6 s-8 d. ouer it, makes 1 l. So that both numbers being added together, the totall shall be equall to the first totall giuen, which prooues the rule infallibly true.

RVLE. III.

Two Numbers being giuen in knowne parts of Coine, to substract the one from the other.

A. BEing indebted vnto *B.* 6857 l--11 s-6 d. hath paid him thereof 5948 l-16 s-8 d. and let it be required to know what is remaining due: First place the two giuen numbers in the Table in their due order, the greatest in the first ranke and the lesser in the second ranke, directly vnderneath, as here they are placed, then beginning (as in this 1 *Rule* of this 2. operation) with the first figures towards the left hand, say, 5. out of 6. resteth 1, wherfore cancel 5 of that 6. and

l.	s.	d.
6857	11	6
5948	16	8

& leaue 1. in the ſame roome; then conſidering the 9. in the next lower roome, cannot bee abated from the 8. ſtanding ouer it, ſay 9 out of 10. reſteth 1. for which 10. cancell the 1. in the next higher roome, and adde the 1. that remaineth to the 8. ouerhead, making together 9; then abate the 4. in the next lower room, out of 5. ſtanding ouer, and there remaineth 1; wherefore cancell 4. of that 5. and leaue 1. in the ſame roome: Next, conſidering that the 8. in the next lower roome, cannot bee deducted from the 7. ouer it, ſay 8 out of 10. reſteth 2. and cancelling the vnit in the next higher room, adde the remainder 2. vnto the 7. which together make 9; then next conſidering the number of *ſhillings* in the abatement, is greater then the number ouer it, ſay 16 out of 20 s. or 1 l. reſteth 4 s. and cancelling a vnite for the 1 l. in the place of vnites, by making the 9. there 8. adde the remainder 4. to the 11 s. in the place of ſhillings, which together make 15 s. and laſtly, in regard that 8 d. cannot be abated out of 6 d. ſay 8 d. out of 12 d. or 1 s. reſteth 4 d. and cancelling a vnite, in the place of ſhillings, adde the 4 d. remaining to the 6 d. which together make

10 d.

l. s. d.	10 d. ſo ſhall your worke
908-14-10	ſtand thus :
5948-16--8	The firſt or vppermoſt of

which two numbers is by the difference or remainder required.

Rvle. IIII.

Two vnequall Inſtrumentall Numbers, being giuen to ſubſtract the one from the other, without fraction, or reduction.

LEt theſe two Numbers 87|640625 bee giuen, whereof the 69|81875 firſt importeth according to ordinarie computation 87$\frac{1}{4}$ & in knowne parts of coine 87l-12s-0d.$\frac{1}{4}$ & the other 69$\frac{13}{16}$, and in knowne parts of coine 69l-16s-4$\frac{1}{2}$d. and let it bee required without fraction or reduction, to ſubſtract the leſſe from the greater. Firſt place both the giuen Numbers in the firſt and ſecond rankes of the Table duely & exactly, as heere they ſtand (for in all workes of this nature, ſpeciall care is to bee had of placing the termes of one and the ſame kinde and Denomination, directly each vnder other, as vnits, primes, and ſeconds, &c. vnder vnites, primes, and

and seconds, &c. The Numbers being thus placed, then next is to bee noted what parallel line passeth betweene the integers and fractionall termes, and marke that out as before for your prime line, and then for operation proceed in all respects as is taught by the 1. *Rule* of this *second* operation for substracting of whole Numbers, wherein there is no difference at all. So shall your remainder be 17321875 that is, $17\frac{163}{320}$ and in knowne parts of coine, 17 l-16 s-$5\frac{1}{4}$ d. which is most speedily wrought without fraction or reduction as was required.

Rvle. V.

To substract fractionall termes out of Integers.

Let these two Numbers be giuen, whereof the first importeth 24. Integers of any kinde or Denomination, and the other this fraction $\frac{251}{320}$ which in knowne parts of coine is 15 s-$8\frac{1}{4}$ d. & let it be required to substract the one from the other. To performe operations of this nature, the best course is first with discretion, to make choice and marke out your prime line, which is to bee chosen in such sort as

24|.....
|784375

there

there may be places ſufficient aboue the ſame line towards the left hand, wherein to place the Integers; and likewiſe beneath towards the right hand, wherein to place the fractional termes; then place the firſt in the higher ranke, in due order, the integers aboue your prime line, and the fractional termes in the ſecond ranke beneath the ſame; wherein ſpeciall care is to be had of due placing the fractionall termes in their naturall & proper rooms; as if the firſt fractionall terme be a *prime*, *ſecond*, or *third*, to place the ſame in the firſt, ſecond, or lower roome from the prime line; and by no meanes to place a *ſecond*, in the place of *primes*, nor a *third* in the place of *ſeconds*, and the like of the reſt: which rightly vnderſtood & obſerued, and the giuen numbers placed, as in the preſent example, conſidering there with all that directly ouer the 7, being the firſt terme of the abatement, there is nothing from whence to abate that 7; beginne your operation thus, ſaying, 7 out of 10 reſteth 3, for which 10 abate 1. out of the 4 in the place of vnites, and place the remainder 3 ouer 7. (for out of this vnite thus abated from the 4, is all the following fracti-

on

onall termes to bee extracted, then substract the 8. *seconds*, saying 8 out of 10. resteth 2, for which 10 abate 1. from the 3 in the place of *primes*, and place the 2 ouer the 8 in the place of *seconds*. And thus proceeding from place to place, substracting each seueral terme of the abatement out of 10, and abating for that 10, an vnite out of the next higher terme in the first ranke, and place the remainder alwaies directly ouer the terme substracted. So in conclusion your worke shall stand thus, the remainder being 23 2 1 5 6 2 5, which is, 23 $\frac{60}{320}$ or in knowne parts of coine 23 l-4 s-3 $\frac{3}{4}$ d. the thing required.

23	2 1 5 6 2 5
	7 8 4 3 7 5

The proofe of Addition and Substraction.

These two operations of *Addition* and *Substraction* (though contrary in kinde) are by *Artists* called *operations by equality*, for that as in Addition, the totall found, is alwaies equall to the seuerall parcels: So in *Substraction*, the remainder found together with the abatement, is euer equall to the first totall, whereon it followeth that either of these operations, is a most true and certaine proofe of the others truth and certainty. For hauing

uing added any two Numbers together, if you againe ſubſtract the one from the other; the remainder ſhall bee your firſt number, if truely wrought. Likewiſe hauing ſubſtracted one number from another, if you adde again the abatement and remainder together, you haue the firſt totall, if truely wrought; which kinde of proofe is moſt infallible, and to bee preferred before that vſuall and ordinary of caſting away Nines, which is moſt erronious, if either by negligence or otherwiſe like certaine numbers of nine bee added or abated in the numbers giuen, as in the numbers required or ſought for.

The properties of Subſtraction.

If equall numbers be ſubſtracted from equall numbers, their remainders ſhall be equall, as from 12 and 12. take 5 and 5, being equall numbers, their remainders ſhall bee 7 and 7. which are alſo equall: but if from equall bee ſubſtracted vnequall numbers; or from vnequall be ſubſtracted equall numbers; their remainders ſhall bee vnequall; as from 6 and 6, which are equal numbers, abate 4 and 3, being vnequall numbers, their remainders ſhall be 2 and 3, which are alſo vnequall; Likewiſe from 8 and 9, being vnequall

equall numbers, take 4 and 4. being equall, their remainders ſhall bee 4 and 5, which are alſo vnequall, *Euc.* 1. *Com. Sen.* 3. 4.

Of Multiplication; the 3. operation Arithmeticall.

Definition.

MVLTIPLICATION, is the vnfolding of any two giuen numbers together; in ſuch ſort, that a third number thereby produced ſhall haue ſuch reaſon and proportion to either of the giuen numbers, as the other of them hath to an vnite: as let 8 and 6. be two numbers giuen, and multiplied the one by the other; you ſhall produce for your third number (being called the *factus* or product) 48, which I affirme to beare the ſame proportion to 8 as 6. doth to 1, and the ſame proportion to 6, as 8 doth to 1. For firſt placing them thus, 48. 8. 6. 1: it plainely appeareth, that as 48 is 6 times 8, ſo

ſo is 6. 6 times 1. Againe, if you place them thus, 48. 6. 8. 1 : now as 48 is 8 times 6 : ſo is 8, 8 times 1. Or if you place them thus, 1. 6. 8. 48, heere, as 1 is to 6, ſo is 8 to 48, or howſoeuer diſpoſed or conuerted (ſo as the product 48 and 1, be made either the two meanes, or the two extreames) each couple ſhall bee of like proportion, which is generall in all Multiplications.

Of the two giuen numbers in this operation, the greater is vſually called the Multiplicand ; and the leſſe the Multiplier, which appellation may bee different to either, for that it is all one thing, to multiply the greater by the leſſe, or the leſſe by the greater, as 3 times 4, or 4 times 3, makes either of them 12.

Before wee enter into the order of working, in regard that the Multiplication of mixt numbers, will at firſt ſeeme troubleſome to the young Practitioner, if he be not formerly perfect in the Multiplication of Digits : I will here preſent you with a ſmall Table of Multiplication of thoſe Digits, which of neceſſitie is to be learned by heart, and well imprinted in minde before you vndertake the work of Multiplication.

The

The Table of Multiplication

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The vse of this smal Table is thus; You are first to consider that all the Digit numbers are heerein twice expressed, that is, first in the head or vppermost ranke in the Table, and againe in the left side thereof; wherefore if you desire to know the product of any Digit number multiplied by another; as if you would knowe the product of 7 times 8, then looke for 7 in the left side, and for 8 in the head of the Table; and following the rankes of those 2. numbers in the square where they meet, you shall finde placed 56, which is the product of 7, multiplied by 8; and the like of all others. And being thus furnished for multiplying of digits, wee will now proceed to practise in operation.

RVLE

Rvle I.

To multiply any Number giuen or required by one terme or figure.

LEt 4987, bee a whole number giuen, and let it be required to multiply the ſame by 6. Firſt place your multiplicand in the firſt ranke of the Table towards the right hand, each terme or figure in order vnder his proper figue, as before is taught; and conſidering that your multiplier conſiſteth but of one terme or figure, it ſhall not be needfull to expreſſe the ſame at all in the Table, but to beare it in minde. And to beginne alwayes your Multiplication at the firſt & higheſt figure towards the left hand: beſides when your Multiplier is but one figure, place alwaies your product or third number ſought for, in the firſt rancke of the Table, where the Multiplicand now ſtandeth, ſaying thus; 6 times 4, is 24, whereof place the 2 in the next higher roome, and let the 4 remaine as before: Then ſay, 6 times 9 is 54, whereof place the Digit 4 where the 9 now ſtandeth, and adde the 5 to the 4 in the next

next higher roome, which together make 9; then to the next lower terme say, 6 times 8, is 48: whereof adde the 4 to the 4 in the next higher roome, which make together 8, and let the 8 last multiplied, remaine as it is in the Table, for the Digit of your product. And lastly, say 6 times 7 is 42, whereof place the 2 where the 7 now standeth (alwaies cancelling the same 7) and adde the 4 to the 8 in the next higher roome, which together make 12; wherefore cancell that 8, and place the Digit 2 of this 12 in the same roome, and the 1 in the next higher roome with the 8 there already placed, making together 9, and your product shal be 29922: the thing required, which operation is most speedily wrought, without pestering the Table with many numbers, and vsing onely one ranke thereof.

Rvle. II.

To multiply any whole Number giuen or required by diuers termes or figures.

LEt these two whole numbers 4632 be giuen, and let it bee required to multiply the one by the other. 24

First

First place the Multipicand in the first ranke, each terme in his due place, and the Muitiplier vnderneath in the second ranke directly, as here they ſtand, and according to the vſuall order with the pen; Firſt multiply each ſeuerall terme in the Multiplicand, by the laſt & loweſt terme of the Multiplier towards the right hand, ſaying, 4 times 2 is 8, which 8 place directly vnder the 4 in the third ranke; then ſay, 4 times 3 is 12, whereof place the digit 2 in the third ranke, directly vnder the 3, and the 1 in the next higher roome; then ſay 4 times 6 is 24, whereof place the digit 4, directly vnder the 6 in the ſame roome, with the 1 there already placed, by cancelling the ſame 1 and opening the 5, and place the 2 in the next higher roome, and then ſay, 4 times 4 is 16, whereof place the digit 6 vnder the 4, with the 1 there laſt placed, which together make 8, and place the 1 in the next higher roome, ſo haue you now finiſhed with your Multiplier 4, which you may either cancell, or let ſtand at your pleaſure, & your worke ſtands thus;

	4632
	24
	18528

Then next are you likewiſe to multiply each terme in your Multiplicand by 2, the other terme in

your Multiplier; saying 2 times 2 is 4, which place vnder 2 your Multiplier, with the 2 there already placed, which together make 6; then say, 2 times 3 is 6, which adde to the 5 standing in the next higher room, to the 6 last placed, which 6 & 5 together make 11; whereof (cancelling the 5) place the digit 1 in the same roome, and the other 1 with the 8, in the next higher room, which together make 9; then say, 2 times 6 is 12, which together with the former 9 make 21, whereof place the 1 in the same roome where the 9 now standeth (by cancelling 8 there) and place the 2 in the next higher roome with the 1 there already placed, which together make 3. And lastly, say 2 times 4 is 8, which added to the last 3, makes 11, whereof place 1 where 2 standeth (by cancelling 2) and place the other 1 in the next higher roome. So shall your worke stand thus; whereof the last number is your product required.

4632	
24	
11168	

It is to bee noted that in this kinde of Multiplication, when you multiply by the first terme of the Multiplier, the seuerall termes of your product are euer to bee placed

ced directly vnder the seuerall termes of the Multiplicand; but when you come to the second terme of the Multiplier, then is euery terme of the product to be placed in one roome higher, then that wherein the terme of the Multiplicand standeth; and so of the rest still increasing higher by one place, as the finished termes of your Multiplier increaseth.

But seeing by this Instrument there is great diuersity and variety in kindes of operation; and that this last rule, and the order of working therein, is alwaies fittest when the termes of the Multiplier are but few; I will here next deliuer a most absolute & easie course of Multiplication, worthy of note & vse; especially when the termes in the Numbers giuen are great or many, or when you are to diuide the product after Multiplication, as is vsuall in the golden rule.

RVLE. III.

To multiply two whole Numbers giuen, the one by the other, vſing onely two rankes in the Table.

LEt theſe two whole numbers be giuen, & let it be required to multiply the one by the other. 867954 65347

Firſt place the Multiplicand in the firſt ranke in the Table, each terme in his proper place, as before is taught; and then the Multiplier in the ſecond ranke, placing alwaies the laſt and loweſt terme thereof, directly vnder the firſt and higheſt of the Multiplicand, in order as heere they ſtand; and in this kinde of operation the product is euer to be placed in the firſt ranke; being the ſame wherein the Multiplicand ſtandeth, ſo ſhall you vſe but two rankes in this whole operation: Now the order of this worke is much different from that in the laſt Rule, or from the order with the pen; for here are you firſt to multiply the firſt terme of the Multiplicand, by all the ſeueral termes of the Multiplier, ſaying, 6 times 8. is 48, whereof place the 8 directly ouer the 6,

and

and the 4 in the next higher roome towards the left hand; then say, 5 times 8 is 40, whereof adde the 4 to the last 8 placed ouer the 6, which together make 12, wherefore cancelling the 8, place the 2 in the same roome, and the 1 adde to the 4, in the next higher roome, making together 5, as for the cypher in the 40, suppose it to stand ouer the last multiplying terme 5, and therefore leaue that place as yet vnaltered: Next say, 3 times 8 is 24, whereof place the 4 directly ouer the 3, and the 2 in the next higher roome; then say, 4 times 8 is 32, whereof place the 2 ouer the 4, & the 3 with the 4 in the next higher roome, making together 7, and next say, 7 times 8 is 56, whereof place the 6 directly ouer the 7, by cancelling 2 of the 8 in the same room, & place the 5 with the 2 in the next higher roome, making together 7: and thus haue you fully finished with the first terme of your Multiplicand. Wherefore now are you to remoue your Multiplier one place lower, towards your right hand, placing the last and lowest terme thereof, vnder the 6 of your Multiplicand, and then shall your worke stand thus. Then begin

5.2277667954
65347

as before, with the first figure of your Multiplier, and that which standeth directly ouer the last terme thereof, saying, 6 times 6 is 36; whereof adde the 6, to the 2 standing ouer the 6 of your Multiplier, and the 3 to the 2 standing in the next higher roome; then say 5 times 6 is 30, whereof adde the 3 to the 8, standing in the next higher roome ouer the 5, your multiplying terme, as for the cypher trouble not your selfe therewith, for it alters not the worke; then next, say 3 times 6 is 18, whereof adde your 8 to the 7, directly ouer the 3, and the 1 to the next higher roome; then say 4 times 6 is 24, which adde to the 6 ouer-head, saying, 24 and 6 makes 30: wherefore cancell the 6 and place three in the next higher roome; and then say 7 times 6 is 42, and cancelling the 6, place the digit 2 in the same room, and the 4 in the next higher roome. And thus haue you finished with the 2 first figures of your Multiplicand: Wherefore againe remoue your Multiplier one roome lower as before, and then shal your worke stand thus; and so working on as before, and multiplying each terme after other

5619842795 4
65347

other of your Multiplicand, by euery terme of your Multiplier, and remouing the same as before, as often as there are termes or figures in your Multiplicand; you shall in conclusion produce this product, 5671819038. the thing required.

This I hold for speed, ease, and certaintie, to be the most absolute operation in great Numbers; the vse whereof is much more easily obtained by smal practise, then many wordes; without which it cannot bee well expressed, and yet seemes thereby more intricate and tedious, then in truth it is, as by practise will soone appeare.

Rvle. IIII.

Two Instrumentall Numbers being giuen to multiply, the one by the other, whereby all fractionall Multiplications are to bee wrought in whole Numbers without fraction or reduction.

LEt these 2 instrumentall numbers be giue; whereof the first importeth $6\frac{25}{89}$ and in knowne parts of coine 6 l–14 s–6 d. and the other $89\frac{75}{89}$ & in knowne parts of coine

6725
8975

89l-15s-0d. and let it bee required in multiplying the one by the other, to produce the product without vse of fraction or reduction or, (which is all one) let this question bee propounded: If one yard or pound of any commodity cost 6l-14s-6d. what cost 85¼. In workes of this nature, hauing placed your numbers, as in the last rule, and as heere they stand: then next are you to finde and marke out your *prime* line, thus; Consider what are the last and lowest fractionall termes in the two giuen numbers, which here in your Multiplicand is *thirds* whose signe is 3, and in your Multiplier *seconds*, whose signe is 2, which 2 signes being added together make 5, and so many places are you to reckon towards the left hand (beginning with the place of vnites) and take the fift place for the place of *primes*: wherefore the next parallel line, aboue that fift place towardes your left hand, shall bee your *prime* line; which rule is generall for finding your *prime* line in all Multiplications of this nature. And being thus prouided of your *prime* line, then proceed with your Multiplication in all respects, as if they were whole numbers, according to the 3. *Rule*

of

of this operation: So ſhall you produce aboue your *prime* line towardes the left hand 603, for ſo many integers, and beneath your *prime* line 56875 *viz.* together 603 56875, which is $603\frac{91}{160}$ and in knowne parts of coine 603 l–11 s–4½ d. being moſt ſpeedily wrought in whole numbers, without fraction or reduction as was required.

Rvle V.

Two fractions being giuen to multiply the one by the other in whole numbers, without fraction or reduction.

LEt $\frac{5}{8}$ and $\frac{7}{40}$ bee two fractions giuen, and let it be required to multiply the one by the other in whole numbers without fraction or reduction; or (which is all one) to finde a third fraction which ſhall bee in ſuch proportion to either of theſe giuen fractions, as the other is to an vnite. Firſt place in the Table your inſtrumentall numbers thus, the Multiplicand in the firſt rank, & the Multiplier in the ſecond; whereof the firſt importeth $\frac{5}{8}$ or in knowne parts of coine, 6 s-3 d. & the other $\frac{7}{40}$, or in knowne parts of coine 3 s-5 d. then find & mark

1 . . .
. 3125
175

out by the 4. *Rule* of this operation, your Prime line, which by the same rule is the 7 parallel line from the place of vnites; which being had, then multiply the one by the other in al respects as if they were whole numbers, as is taught by the 2. *rule* of this operation, whereby you shal produce $\frac{7}{46875}$ which importeth $\frac{7}{128}$ or in knowne parts of coine 1 s-1 $1\frac{1}{2}$ d. the product required.

Herein is to bee noted two speciall points most worthy obseruation.

First, that a fraction being multiplied by a fraction, the product is euer lesse then either of the giuen numbers; which Rule is general.

And secondly, that notwithstanding your Prime line in these operations, hath no vse to distinguish integers frō fractions, for that the whole operation consisteth of fractionall termes; yet it serueth to this excellent purpose, at the first view, to shew and declare the value of the fractionall termes in the product. For hauing wrought your multiplication, you finde the first terme 5 of the product, to stand in the second lower roome to your prime line, which is euer the place of seconds, & cōsequētly the following terms are thirds, fourths, &c. in their naturall order.

Diuers

Diuers Briefe and ready Rules, for ſpeedy operation.

RVLE VI.

To reduce pounds into ſhillings, or to multiply by 20.

LEt 364 l. bee a number giuen, and let it bee required to know how many ſhillings are therein: Firſt place the giuen number, as before, and then by *Rule* 1. of this operatiõ multiply the ſame by 2, but place alwaies the product in the next higher roome: as thus, 2 times 3 is 6, cancell the 3 and place the 6 in the next higher roome towardes your left hand, being the place of thouſands; then ſay, 2 times 6 is 12, & cancelling the 6, place the digit 2 of your 12, in the next higher roome to the place wherein the 6 ſtood, and the 1 with the 6 in the next higher roome to that; then laſtly, ſay 2 times 4 is 8, and cancelling your 4, place the 8 in the next higher room, to that wherein the 4 ſtood,

4 ſtood. So haue you 7280, for the number of ſhillings in 364 l. the thing required.

Whereby alſo it appeareth that if you would multiply any number giuen, by 30. 40. 50. 80, &c. it is but multiplying the ſame in the next higher roome, by 3. 4. 5. 8, &c.

Or if any number be to bee multiplied by 10. 100. 1000, &c. it is but remouing alwayes the giuen number, ſo many places higher towardes the left hand, or the prime line ſo many places lower, as there are cyphers in the multiplier.

Likewiſe by this Rule it is apparant that if any number were to bee multiplied, by 300. 5000. 80000, &c, it is but multiplying the giuen number by 3. 5. or 8, &c. ſo many places higher, as there are cyphers in the multiplier, or remouing the Prime line ſo many places lower, which is all one thing.

RVLE. VII.

To reduce ſhillings into pence, or to multiply by 12.

LEt 463 s. be a number giuen, and let it bee required to know how many pence

pence are therein : First hauing placed your giuen number in his due place, then according to *Rule* 1. of this operation, multiply each terme therein by 6, and the product ſhall bee 2778, which product doubled will bee 5556, the number of pence in the giuen number, as was required.

By this Rule may alſo be gathered, that if it bee required to multiply any number by 14. 16. *or* 18, *it is but to multiply the ſame, by* 7. 8. *or* 9, *being* $\frac{1}{2}$ *of the multiplier, and doubling the product.*

RVLE. VIII.

To reduce pounds into pence, or to multiply by 240.

LEt 34 l. pound bé a number giuen, and let it be required to know how many pence are therein. By the firſt Rule of this operation, multiply the giuen number 34 by 6, the produét will bee 204; which product by *Rule* 6 of this operation, multiply by 4 in the next higher roome, whereby ſhall bee produced 8160, the product required.

RVLE

RVLE. IX.

To multiply any number by 5.

LEt it bee required to multiply 546 by 5. First plaee the giuen number a roome higher, or the Prime line a roome lower, so is it 5460, whereof take halfe, which is 2730, for the product of 546, multiplied by 5, as was required.

RVLE X.

To multiply any number giuen by any aliquot part of 100. 1000, &c.

LEt it be required to multiply 3647 by 25, & considering that 25 is $\frac{1}{4}$ of 100. First place the giuen numbers 2. roomes higher, or remoue the Prime line two roomes lower, that is as many roomes as there are cyphers in 100, so is it 364700, whereof take $\frac{1}{4}$ which is 91175. for the product of 3647, multiplied by 25, the thing required, and the like of all others.

RVLE

Rvle XI.

To multiply any number giuen by 9.

LEt it be required to multiply 532 by 9, place the giuen numbers a roome higher, so is it 5320, from whence abate the giuen number it selfe, the remainder is 4788, being the product of 532 multiplied by 9, the thing required.

Rvle XII.

To multiply any number giuen by shillings.

LEt it be required to know the value of 4653 yards, or pounds of any commodity, at 16 s. the yard, or pound. First place your giuen number in any part of the Table (so as you reckon the next parallel after the lowest terme of the giuen number for *prime* line) then multiply the giuen number by 8 (that is, alwaies by halfe the number of shillings giuen) in the next lower roome, or (which is all one thing) in the same roome (taking the next higher parallel for *prime* line.)

So

So ſhall your product be 37221-8ſ-0d. But if the queſtion were made of any odde number of ſhillings, as 17ſ. then are you to multiply (as before) by 8½ that is, firſt by 8, and taking that product, & halfe thereof, you haue 5583 l-12ſ-0d. your deſire.

Rvle XIII.

To finde the true value in knowne parts of coine, of any fractional termes, in Inſtrumentall operations.

SVppoſe that after any Inſtrumentall operation, you find in the Table theſe fractionall termes $\frac{40}{62}$, and for want of practiſe, skill, or memory, you know not the true value thereof. Firſt conſider, that the firſt greateſt terme towardes the left hand, is leſſe then 5 *ſeconds*; and therefore the whole value of theſe fractionall terms leſſe then 12d. Wherfore by *rule* 8, of this operatiõ multiply thoſe fractional termes ſo found or giuen by 240, which are the number of pence in 1 l. and your product ſhall bee 97½, whereof the 9 Integers or vnites (which you ſhall euer note aboue your *prime* line) is ſo many pence,

pence, and the fractionall termes remaining appeare at the first to bee the $\frac{1}{4}$ of a penny: But supposing you knew not the value of this or the like remainder, multiply the same alwaies by 4. because 4. farthings makes a penny, and you shall heere finde for your product, 3. vnites standing aboue your *prime* line, and nothing remaining: wherefore I conclude that the fractionall termes first found or giuen, are in value $9\frac{3}{4}$ d. the thing required, and the like of all others.

If any remainder were after the multiplication by 4, the same should haue beene the fractionall parts of a farthing.

If the greatest fractional terme in the number so found or giuen, had beene more than 4. seconds, as suppose it had beene, 79 6: 5: then must you according to Rule 6. haue multiplied the same by 20: whereby you should haue produced aboue your prime line 15, for so many shillings: Or rather (seeing it is apparent that 7. Primes, and 5 seconds make 15 d.) first to abate those 7. primes, and 5 seconds, and to worke for the rest as before.

To finde the precise quantity or value of such fractionall termes, as they are any part or parts of an vnite: shall be taught

in

in the next operation, being wrought by abreuiation or diuision.

Certaine properties of Multiplication.

If any number bee a Multiplier of diuers numbers, and likewise of their totall added together, the seuerall products of those first multiplications being added together, shall be equall to the product of their multiplied totall. As let 3 be a multiplier of 4. 6, and 8, their seuerall products being 12. 18. and 24, which together make 54. I affirme if I likewise multiply the totall of 4. 6. and 8, which is 18, the product shall bee likewise 54. *Euc. 2. 2. Barl. ibidem.*

Any 3 vnequall numbers being giuen, if the meane number bee multiplied by the totall of both the differences, whereby the same mean differeth from the two extreames; the product shall be equall to both the products, made by the multiplying of each extreame, by the others difference, as let 8. 12. and 17, bee 3 vnequal numbers giuen: now the difference betweene 8, the lesser extreame, and 12 the meane, is 4, and the difference betweene 12 the meane, and 17 the greater

ex-

extreame, is 5; the totall of which differences make 9, whereby let 12 the meane be multiplied, the product is 108, which I affirme shall be equall to both the products 8 multiplied by 5, & of 17 multiplied by 4, being either extreame multiplied by the others difference from the meane; which are likewise together 108. *And hereon dependeth the maine ground and reason of the Rule of Alligation.*

If any one number multiply two numbers giuen, the numbers produced by such Multiplication, shall haue such proportion as the multiplied numbers haue. As let 5 and 8 be two numbers giuen, & let them bee seuerally multiplied by 3, whose products shall be 15 and 24. Now I say, that 15 hath the same proportion to 24, as 5 hath to 8; wherefore 5, 8. 15. and 24, are 4 proportionals, *Euc.* 7. 17.

The proofe of Multiplication.

The best and surest proofe of Multiplication is by Diuision, and of Diuision by Multiplication: wherefore wee will speake further heereof in the conclusion of the next.

Of

Of Diuision, the fourth Operation Arithmeticall.

Definition.

DIVISION, is an operation Arithmeticall, whereby is found out how often any leſſe number is contained in the greater, or what part or parts any leſſe number is of a greater.

In this operation, there are alwayes 3 principall numbers, whereof two are giuen, and the third is required or ſought for.

Of the two giuen numbers the greater is called the Diuidend, and the leſſe the Diuiſor, if the leſſe diuide the greater: but if the greater diuide the leſſe, then is the leſſe number called the Diuidend, and the greater the Diuiſor; the third number in this operation (being that which is required or ſought for) is called the *quotus* or *quotient*, becauſe it alwayes ſheweth how many times the

leſſe

leſſe number giuen is contained in the greater, or what part or parts the ſame leſſe number is of the greater.

Rvle. I.

Two vnequal whole numbers being giuen, to diuide the greater by the leſſe.

LEt theſe two vnequall numbers bee giuen, 15168. and 24, and let it bee required to diuide the greater by the leſſe. Firſt place your dividend, 15168, in the firſt ranke of the table, and in the higheſt roomes thereof towardes your left hand, that is, the firſt terme or figure 1 of your Diuidend in the firſt and higheſt roome of the Table, and the reſt following in order; Then place your Diuiſor 24, euer vnderneath your Diuidend in the ſecond ranke: and conſidering that 2. the firſt terme of your Diuiſor, is greater then 1, the firſt terme of your Diuidend; you are to place your Diuiſor a roome lower towards the right hand, wherefore place the firſt terme 2. of your Diuiſor, directly vnder the ſecond terme 5. of your Diuidend, and the 4. of your Diuiſor in the next lower roome, that is, directly vnder the

the 1. in the middle of your Diuidend: So shall your numbers stand in the Table thus: Then marke out your Prime line, which (in diuiding of whole numbers, whether you diuide the greater by the lesse, or the contrary) is euer that parallel line which followeth next after the last & lowest terme of your Diuidend, whether it bee a cypher or signifying figure. And now are you fit for operation, wherein first you are to seeke how often your Diuisor 24, is contained, in so much of your Diuidend as standeth ouer it, and in the next higher roome towardes the left hand, which is heere 151: but because it were somewhat troublesome, alwayes to finde out how often the whole Diuisor is contained in that part of the Diuidend ouer it, seeke only how often 2, the first terme of your Diuisor is contained, in so much of the Diuidend as standeth ouer it, and in the next higher room which is here 15, saying how many time 2 is there in 15; which presently appeareth to be 7, and 1 remaining, which 1 together with the 1 standing ouer the 4, makes but 11, wherein their cannot bee 7 times 4 (the other terme of your Diuisor) and therefore the whole

15168	
24	

whole Diuiſor 24 is not contained 7 times in the 151 ſtanding ouer it; wherefore take one leſſe, ſaying that 7 is contained in 15. 6 times, which 6. is the firſt terme or figure in the *quotient*, and is euer to bee placed in the third ranke directly vnder the laſt terme or figure of the Diuiſor, & your worke ſhall ſtand thus: then are you next to multiply each terme in your Diuiſor by 6, the laſt terme found, and to deduct the ſeuerall prodncts from your Diuidend, ſaying thus, 6 times 2 is 12, out of 15 reſteth 3, and cancelling the 15, place the 3 directly ouer the 2, in the roome where the 5 now ſtandeth; Then ſay 6 times 4 is 24 out of 30 (naming alwaies that Article number which is next aboue your preſent product, as 10. 20. 30. 40. 50, &c.) reſteth 6, wherefore cancelling the 3 now ſtanding ouer the 2, the firſt terme of the Diuiſor, and place the 6 with the 1, directly ouer the 4. So haue you wrought your firſt operation, wherefore now are you to remoue your Diuiſor a place lower towardes the right hand, and your worke ſhall ſtand thus. Then aske againe how many times 2. (the

15	168
24	
6	

7	68
24	
6	

first

first terme of the Diuisor) is there in 7, which is thrice or 3 times, & 1 remaining, which, with the 6 ouer the 4, maketh 16; and it is also apparant that 4 (the other terme of the Diuisor) is likewise contained thrice or 3 times in 16; wherefore take 3 for the second terme in the *quotient* and place it directly vnder the 4, the last terme in the Diuisor; then multiply the Diuisor by that 3, and abate the product from the Diuidend, saying thus; 3 times 2 is 6, out of 7 resteth 1, wherefore cancell 6 of the 7 and leaue 1 in the same roome, then say, 3 times 4 is 12, out of 16 resteth 4, and cancelling the 16 leaue 4 in the roome where 6 now standeth; So haue you wrought your second operation; and now are you againe to remoue the Diuisor a place lower towards the right hand & your worke shall stand thus. Then aske againe how many times 2 there is in 4, which is twice, or 2 times; and it is likewise apparent that 4, the last terme of the Diuisor, is contained twice, or 2 times in 8; wherefore place 2, the third terme in the *quotient*, directly vnder 4, the last in the Diuisor, then multiply the Diuisor by that 2, and abate the product from

48
24
63

from the Diuidend, saying thus, 2 times 2 is 4, out of 4 resteth nothing; wherefore cancell the 4 in the Diuidend, and then say 2 times 4 is 8; out of 8 resteth nothing, and cancelling 8 in the Diuidend, there is nothing remaining. So shall the finished work stand thus; whereby it appeareth that 24, the lesse number giuen is contained in 15168 the greater number 632 times; being the *quotient* required, or the third number sought for.

24 | 632

As in all Multiplications it is fittest to place the giuen numbers in the lower roomes of the Table towardes the right hand, because the operations thereby increase towards the left: So in all Diuisions it is meetest to place the giuen numbers in the higher roomes of the Table towards the left hand, for that the operations thereby increase towardes the right, the conuenience whereof in practise will soone appeare.

The Prime line in the former Rule seemeth to serue to little purpose, by reason there was no remainder in that operation: yet is it there a bound or limit, shewing how far your worke is to extend: for in whole numbers the quotient *neuer exceedeth*

that line ; beſides it ſheweth before you begin to worke, how many whole numbers ſhall bee in your quotient ; *for the Diuiſor being placed in his due place (as before) conſidering that alwayes the firſt terme of the* quotient, *is to be placed vnder the laſt of the Diuiſor ; and the laſt terme of the* quotient *(whether it bee cypher or ſignifying figure) is to be placed in the next higher roome to the Prime line ; it is apparant how many termes ſhall bee therein. Beſides which, there are many other excellent vſes of this line ; as well in whole as fractionall operations, as by vſe and practiſe will better appeare.*

A ſecond Example of the former Rule.

LEt theſe two vnequall numbers bee giuen, 764300. and 387, and let it be required to diuide the greater by the leſſe : Firſt place them in the Table as before is taught; with this onely difference, let here the firſt terme of the Diuiſor, be placed vnder the firſt of the Diuidend, becauſe the three termes in the diuiſor are leſſe in value, then the 3. firſt termes of

of the Diuidend: So shal they appeare in the Table thus: Then mark out the Prime line which shall bee the third lower line next after the signifying figures of the Diuidend; including the two cyphers (for all the roomes in the Table being eclipsed, are alike without difference or distinction from cyphers.) And now are you fitted to proceed, asking thus, how many times 3 in 7, which is 2 times; but then consider that 2 times 387, is more then 764 standing ouer it, as plainely appeareth; and therefore say 3 is in 7. 1 time: which 1 place for your first terme of your *quotient*, directly vnder the 7, being the last of your Diuisor; and seeing this figure 1, will neither multiply nor diuide, you are heere onely to deduct each terme of the Diuisor, out of those standing ouer them, leauing their seuerall remainders, saying thus, 3 out of 7 resteth 4, & cancell the 7, or 3 of the 7, and leaue 4 there, then because 8 is to great to be taken out of 6, say 8 out of 10 resteth 2: for which 10 cancell 1 of the 4 in the next higher roome, & place the 2 which remained, with the 6, making the same 6 to be 8: and then say 7 out of 10 (be-

764300.	
387	

cause it will not bee taken out of 4) resteth 3, which 3 place with the 4, making it 7; & the for 10 cancell 1 of the 8 in the next higher roome: So haue you wrought your first operation; wherefore now remoue the Diuisor a place lower, towards the right hand, and your work shall stand thus. Then againe aske how many times 3 in 37, which is 9 times (for more then 9 must you neuer take or expresse) then placing the 9 in the *quotient* vnder der the last terme of the Diuisor, say 9 times 3 is 27, out of 37 resteth 10, for which 27 cancell the 7 ouer the 3 of the Diuisor, and likewise 2 of the 3 in the next higher roome, and so will there 10 remaine; then say 9 times 8 is 72, for which 2 cãcel 2 of the 7 directly ouer the 8, and for the 7, seeing there stands a cypher in the next higher roome (from whence it should be taken) say 7 out of 10 resteth 3, which 3 place where the cypher stood, & for the 10 cancel the 1 in the next higher roome, and then say 9 times 7 is 63, for which 2, cancell the 3 standing ouer the 7, and for the 6, seeing there is but 5 in the next higher roome, say 6 out of 10 resteth 4, which 4, place

377300
3,87
1

with

with the 5, and for the 10 cancel 1 of the 3 in the next higher roome; So is your second operation finished, and now again remoue the Diuisor a place lower, and your worke shall stand thus. Then againe aske how many times 2 in 29, which will bee but 7 times (in respect of the others greatnesse) which 7 place in the *quotient* vnder the last terme of the Diuisor; and then say 7 times 3 is 21, for which cancell 1 out of the 9, and likewise the 2 in the next higher roome; then say, 7 times 8 is 56, out of 60 (being the next greatest Article number) resteth 4, which 4 place ouer the 8, and abate or cancell 6 out of that 8, in the next higher roome, leauing 2 there, and then say 7 times 7 is 49, out of 50 (being the next greatest Article number) resteth 1, which 1 place directly ouer the 7, & for the 50 (which is 5 in the next higher roome, which cannot be taken out of the 4 there) say 5 out of 10 resteth 5, which 5 place with the 4 making together 9, and for the 10 cancel 1 out of the 2 in the next higher roome. So is your third operation finished, and now againe for the last time remoue the Diuisor a roome lower (which is now

29000	
387	
19	

come to the Prime line, & therefore the last remoue) and your worke will stand thus. Then againe aske how many times 3 in 19, which (in respect of the others greatnesse) is but 4 times; which 4 place in the *quotient*, vnder the last terme of the Diuisor; And then say, 4 times 3 is 12, out of 19 resteth 7, wherefore cancell 2 of the 9, and likewise the 1 in the next higher roome, so will there bee 7 remaining; then say, 4 times 8 is 32, out of 40 resteth 8, which 8 place with the 1 ouer 8 the Diuisor, making together 9, and for the 40 cancell 4 out of 7, in the next higher roome where resteth 3. And lastly, say 4 times 7 is 28, out of 30 resteth 2, which 2 place ouer the 7, and for the 30 cancel 3 out of 9, in the next higher room; so is your diuision fully finished, and your worke standeth thus. Whereof the third and last number is the *quotient* of Integers; and the first and second together make the fraction remaining: wherefore 764300, being diuided by 387, the *quotient*, or third number fought for is $1974\frac{362}{387}$ the thing required.

1910
387
197
362
387
1974

These two former examples being diligently practised

practiſed and well vnderſtood; all the following Rules of this fourth operation will appeare moſt plaine and eaſie: and therefore I purpoſe in their ſeuerall conſtructions, to be as briefe as conueniently I may.

It is to be noted that in this laſt example, the operation being finiſhed, there is remaining a fraction as $\frac{361}{387}$ *which vſually happeneth in moſt Diuiſions, and oftentimes it is requiſite to bee knowne, what ſuch fractions import in the knowne parts of coine; the performance whereof is taught in the following Rule.*

RVLE. II.

Any fraction being giuen, or remaining after Diuiſion to reduce the ſame into ſhillings, pence, &c.

LEt the fraction in the concluſion of the laſt example bee giuen, and let it be required to know the value thereof in knowne parts of coine. Firſt ſuppoſe the ſame to bee ready placed in the Table, as in concluſion of the former worke, where

it ſtandeth in order thus ; each ſeuerall number being limitted and bounded towardes the right hand with the Prime line, whereof, as before, the firſt number is the Numerator of the Fraction, the ſecond the Denominator, and the third the *quotient* of Integers, which vnderſtood worke thus, according to *Rule* 6 of Multiplication, multiply the Numerator 262, ſo is the ſame reduced into ſhillings, and your worke will ſtand thus ; then remoue your Denominator 387 (which is ſtill to bee your Diuiſor, as it was in finding the Integers in the laſt example) one roome higher towards the left hand, that is the 3 vnder the 7, and the reſt in order ; and being ſo placed diuide thereby your Diuidend 7240, as was taught in the laſt example, placing the termes of your *quotient* beneath the Prime line in the ſame ranke, with the Integers formerly had ; ſo ſhal you produce in your *quotient* 18, which are ſo many ſhillings, and your worke ſhall ſtand thus : The firſt number being Numerator, and the ſecond Denominator of the fraction of a ſhil-

362	
387	
1974	
7240	
387	
1974	
274	
387	
1974	18

ling

ling, and the 3 the *quotient* of pounds and ſhillings: now to reduce that fractiō into pence, multiply the Numerator 274 as it ſtands by *Rule* 7 of Multiplication; and your worke ſhall ſtand thus: and now without remouing your Diuiſor, diuide this 3288 by 387, ſo ſhall you produce in the *quotient* 8, which are ſo many pence, and are to bee placed in the ſame ranke next after the 18 s. and your work ſhall ſtand thus: So is the whole *quotient* of this and the former Diuiſion together 1974l–18s–8d. $\frac{192}{387}$ the thing required. And if you are deſirous to know how many farthings this laſt fraction importeth by *Rule* 1 of Multiplication, multiply the Numerator 192 by 4, and diuiding as before, you ſhall produce $\frac{1}{4}$d. and $\frac{188}{387}$ of $\frac{1}{4}$.

3288	
387	
1974	18

192	
387	
1974	18. 8

But it is to be vnderſtood, that without this reduction, the preciſe value of the former fraction is moſt eaſily had in the knowne parts of coine by Inſtrumentall operation in whole numbers, as ſhall be taught in the next.

RVLE. III.

Two vnequall whole numbers being giuen, and diuide the greater by the leſſe, and hauing a fraction remaining after ſuch Diuiſion, to finde the true value of the ſame fraction, in knowne parts of coine by operation in whole Numbers.

LEt the whole numbers giuen, and diuided, be thoſe in *Example* 2. *Rule* 1 of this operation, and let the fraction remaining be that in the concluſion of the ſame Diuiſion, where the numbers ſtand thus: whereof the 2 firſt are the fraction remaining, and the third and laſt the *quotient* of Integers: and let it be required to finde the true value of the ſame fraction in knowne parts of coine, by operation in whole numbers: As by the ſecond example of the firſt *Rule* you obtained thoſe Integers 1974, ſtanding aboue the Prime line, ſo are you to worke forward in all reſpects (as if your worke were vnfiniſhed), with the ſame diuiſor and remainder (without alteration) for obtaining the *Primes, ſeconds, thirds*, &c.

$$\frac{362}{387}\ 1974$$

to

to bee placed beneath the Prime line: whereby at firſt view ſhall appeare what ſhillings, pence, &c. that fraction importeth thus. Firſt remoue your Diuiſor 387, a place lower towards the right hand, ſo ſhall the loweſt terme thereof bee in the place of Primes. And conſequently the firſt found terme in this new *quotient*, ſhall be ſo many Primes, for that it is alwayes to bee placed vnder the laſt and loweſt terme of the Diuiſor, and your Diuiſor being ſo remoued, the worke ſhall ſtand thus: Then ſay, how many times 3 in 36, which is 9 times, and placing the ſame in the place of Primes, vnder the laſt terme of the Diuiſor, ſay 9 times 3 is 27, out of 30 reſteth 3, which 2 place with the 6, ouer the firſt terme of the Diuiſor, and for the 30 cancell the 3, in the nexr higher roome; then ſay, 9 times 8 is 72, for which cancell the 2 ſtanding ouer the 8, and 7 out of 9 in the next higher roome; then againe, ſay 9 times 7 is 63, out of 70 reſteth 7, which 7 place ouer the laſt terme of the Diuiſor, and ſeeing that for the 70 you ſhould abate 7 out of the next higher roome, where is nothing but a cypher, therefore ſay, 7 out

362	
38	7
1974	

of 10 resteth 3, which 3 place where the cypher stood, and for the 10 cancell 1 of the 2 in the next higher room. So is your first operation finished, & your worke standeth thus. Whereby you haue already gotten 9 *Primes*, being equall to the 18s. found by 2 seuerall operations in the former *Rule*. And now againe are you to remoue your Diuisor a place lower, towardes your right hand, and working in all respects as before, you shall finde your worke stand thus; whereby you shall produce in *quotient*, 3 *seconds*, which is $7\frac{1}{3}$d. Then the third time remoue your Diuisor a roome lower, and worke on exactly as before, and you shal finde your worke standing thus: whereby you haue now gained in *quotient* 5 *thirds*, that is, $1\frac{2}{3}$d. wherefore we haue now already by the worke of this *Rule* 18s-$8\frac{1}{3}$d. But yet for more exactnesse once more remoue the Diuisor, and worke as before, till your worke stand thus; where now you haue gotten more in *quotient*, then

13		7
38		7
1974		9
2		09
3		87
1974		93
		155
		387
1974		935
		2.
		387.
1974		9354.

then

then before 4 *fourths*, being a terme of ſo ſmall a value, that euery vnite therein, is very little more then halfe a Mite; as I ſpake before in my Notations, & yet is there here a remainder left of $\frac{3}{387}$ of an vnite in this place, or $\frac{1}{387}$ of $\frac{1}{1000}$ of an vnite in the loweſt place of Integers.

And thus by operation in whole numbers, with great eaſe and preciſe exactneſſe, haue wee obtained the true value of the fraction giuen in knowne parts of coine, as was required.

Herein is to be noted that indiuers operations, if you ſhould continue your worke infinitely, yet would there euer bee ſome ſmall fraction remaining; but of ſo ſmal a value, that after you haue wrought to fourths or fifts, it were needleſſe to worke further therein, but to accept of it as a ſufficient preciſenſſe in any operation whatſoeuer.

RVLE. IIII.

Two vnequall whole Numbers being giuen to diuide the leſſe by the greater: Or (which is all one thing) any fraction being giuen to reduce the ſame, into the termes of primes, ſeconds, thirds, &c.

LEt 7 and 16 be two vnequall whole numbers giuen, and let it be required to diuide the leſſe by the greater, or which is the ſame thing, let $\frac{7}{16}$ be a fraction giuen, and let it be required to reduce the ſame into *Primes*, *ſeconds*, &c. Firſt place the giuen numbers towardes the higheſt roomes in the Table, as before is taught thus. Then marke out your Prime line, which is euer that parallel line next after the laſt and loweſt terme of the Diuidend: and now proceed to your worke asking how many times 1 in 7, which will bee but 4 times, which euer place vnder the laſt terme of the Diuiſor, being heere the place of primes; then ſay, 4 times 1 is 4, out of 7 reſteth 3, wherefore cancell 4 of the 7, leauing 3 in the ſame roome; Then ſay 4 times 6 is 24, out of 30 reſteth 6, which 6, place ouer the laſt terme of the Diuiſor, and for the 30 cancell the 3 in the

7	
1	6

the next higher roome. And now is your firſt operation done, whereby you haue gotten in your *quotient* 4 *primes*, or 8 ſhillings, and 6 is yet remaining, wherefore remooue your Diuiſor a place lower, and your worke will ſtand thus. And then againe working in all reſpects as before, you ſhall at this ſecond operation, produce in the *quotient* 3 *ſeconds*, and 12 is ſtill remaining. Wherefore the third time remoue the Diuiſor, and your worke in the Table wil ſtand after this manner. And then againe working the third time directly as before, you ſhall finde in the *quotient* 7 *thirds*, and now is there 8 remaining, and therefore againe remoue the Diuiſor as before, and your worke will ſtand thus. And againe working the fourth time as formerly you wrought, you ſhall produce the *quotient* 5 *fourths*, and now is there nothing remaining; wherefore your worke is fully finiſhed, and thus it ſtandeth. Whereby the leſſe number giuen, is diuided by the greater, and the giuen fraction $\frac{7}{16}$ reduced into 4 *primes*, 3 *ſeconds*, 7 *thirds*, and 5 *fourths*;

6
16
4

12
16
43

8
16
437

16
4375

5 *fourths*; which appeareth at first view to bee together, 8 s. 9 d. which is $\frac{7}{18}$ of 20 s. or one pound, the thing required.

And hereby it is apparant that any number giuen may bee speedily diuided, into any number of equall parts required, whether lesse or greater then the giuen number, without fraction or reduction. As supposing the giuen number 7, in this former rule were 7 l. to bee diuided amongst 16 men, each mans part thereby is found to be 8 s 9 d. being wrought without fraction or reduction.

Rvle V.

To reduce Instrumentall numbers, as primes, seconds, &c. into their least proportionall termes: or (which is one thing) to reduce them into ordinary fractions in their lowest termes.

LEt the *quotient* in the conclusion of the last *Rule*, be an Instrumental numgiuen, and let it be required to reduce the same into the least proportionall termes, or into an ordinary fraction. First let the giuen number bee placed in the Table, as

it standeth in the conclusion of the last worke thus. Then considering that this number (and all other of the like nature, as I haue formerly spoken in my Notations) may bee taken and reputed as the Numerator of a fraction, whose Denominator is so many cyphers as there are termes in the number giuen, with an vnite before them thus. Let there be an vnite opened in the place of vnites, in the next ranke beneath your giuen number, & then shall your worke stand thus. Now are you next to consider by what terme or figure, both your Numerator and Denominator will be diuided or abreuiated, which at first view appeareth to be no other but 5, because that 5 is the last terme of the Numerator; and considering that 5 is not contained in 4, the first and highest terme of the Numerator, beginne your worke thus, asking how many times 5 is in 43, which is 8 times, and 3 remaining; which 3 remaining keepe in minde, and cancelling both the 4 and the 3, place your 8 where before the 3 stood: Now the 3 in mind & the 7 following make 37, wherefore ask how many times 5 in 37 which is 7 times,

4375

4375
10000

4375
1

7 times, and 2 remaining, ſo is your 7 there already placed; and the 2 remaining with the 5 following, make 25, wherein at firſt it appeareth 5 is cõtained 5 times, which is likewiſe there already placed. And ſo you haue once abreuiated your Numerator, and now muſt perform the like by the Denominator; ſaying 2 times 1 is 2, and cancelling the 1, place the 2 in the next lower roome (for to multiply by 2 in the next lower roome, is all one to diuide by 5, as ſhall hereafter be taught; for if you had asked how many times 5 in 10, you had likewiſe found the ſame 2.) And thus haue you ended your firſt operation, and your worke ſtandeth thus. Then the ſecond time to your Numerator, asking how many times 5 in 8, which is 1 time and 3 remaining, wherefore cancell the 8, and place 1 in the ſame roome, and for the 3 remaining, and the 7 following, aske how many times 5 in 37, which is 7 times, and 2 remaining, which 7 is already placed, and likewiſe the next 5, for it appeareth (as before) that 5 is in 25, 5 times without remainder, and therefore the Numerator is abreuiated the ſecond time. Now againe do

$$\frac{875}{2000}$$

like-

likewiſe by the Denominator, ſaying 2 times 2 is 4, which place in the next lower roome, and cancell the 2, ſo haue you finiſhed with both the ſecond time, and thus ſtands the worke. And proceeding thus twice more with both numbers (for both Numerator and Denominator are to be abreuiated by equall termes, and equall times, or the worke is falſe.) You ſhall produce the fraction $\frac{7}{16}$, which are not onely the leaſt propotionall termes of the Inſtrumentall number giuen, but an ordinary fraction equall thereunto in value, as was required. $\frac{175}{400}$

And heere is to be noted, that eyther operation in the two laſt precedent Rules, are infallible proofes and trialls of the others truth: for as by the fourth Rule, $\frac{7}{16}$ the fraction there giuen, produced this Inſtrumentall Number 4375. So by this fifth Rule the ſame Inſtrumentall number giuen, hath heere againe produced the ſame fraction $\frac{7}{16}$.

Now ſeeing there is no difference at all in this fourth operation, betweene the manner and order of whole numbers formerly

ly taught, & those of Instrumentall Diuisions, nor any difficulty at all, other then the finding out of the Prime line: wherby the Integers and fractionall termes in the quotient, *are euer as readily distinguished the one from the other as the worke is finished. I will next heere deliuer an infallible Rule, for the ready finding out of this line in any Instrumentall Diuision whatsoeuer.*

RVLE

Rvle. VI.

To finde out the Prime line in all Instrumentall Diuisions.

In all Diuisions of this nature, whether the lesse diuide the greater, or the contrary, the signes of the Diuidend are euer either equal to those of the diuisor, or else they are greater or lesse then the same: If

- *Equall*: Take alwaies that parallel line, which is next after the last and lowest terme of the Diuidend for the Prime line.
- *Greater*: Substract alwaies the last signe of the Diuisor, frō the last of the diuidend, & the remainder shalbe the signe of that terme in the *quotient*, which standeth directly vnder the last terme of the diuidend, wherby the Prime line doth presently appeare.
- *Lesse*: Adde so many places more (towardes the right hand) vnto the Diuidend, as the last signe of the Diuisor, exceedeth the last signe of the Diuidend, and the line next following those places so added, shall be the Prime line.

Of which 3 seuerall kindes, obserue these 3 following Examples.

Exam-

EXAMPLE I.

Two Instrumentall Numbers being giuen, (whose fractionall signes are equall) to diuide the one by the other without fraction or reduction.

LEt 451575 and 16725 be two Instrumentall numbers giuen, the first importing $451\frac{39}{40}$, or in knowne parts of coine, 451l-11s-6d. and the other, $16\frac{39}{40}$, or in knowne parts of coine, 16l-14s-6d. whose signes are equall, the greatest of either being 3, and let it bee required to diuide the one by the other without fraction or reduction. First place your giuen numbers in the Table, as before is taught, appearing thus; then marke out your *prime* line, which (considering that the signes in both giuen numbers are equall) by the first branch of this present *Rule*, shal be euer that next following after the last and lowest terme of the Diuidend. And then worke on your Diuision in all respects, as before is taught for whole numbers, without any difference at all: So shall you produce in *quotient*

451	575
16	725

27 stan-

27, ſtanding aboue your *prime* line, being therfore ſo many integers, or pounds in coine without remainder; which is the third number ſought for, and the operation wrought without fraction or reduction, as was required.

Example II.

Two Inſtrumentall numbers being giuen, (the fractionall ſigne of the Diuidend, being greater then thoſe of the Diuiſor) to diuide the one by the other without fraction or reduction.

LET 256375 and 6423 bee two Inſtrumentall numbers giuen, whereof the firſt (being the Diuidend) importeth $24\frac{51}{100}$, or in knowne parts of coine 24 l–12 s–3 d. and the other (being the Diuiſor $64\frac{23}{100}$, or in knowne parts of 64 l-4 d-$7\frac{1}{5}$ d. of which two giuen numbers, the ſignes of the Diuidend are greater then the ſignes of the Diuiſor, and let it bee required to diuide the one by the other without fraction or reduction. Firſt place the two giuen numbers in the Table, after the vſuall manner, which will there ſtand thus. Then conſidering the greateſt

256375
6423

ſigne

ſigne of the Diuidend is 4, and the greateſt of the Diuiſor 2, ſubſtract 2 out of 4, and the remainder is 2, which by the ſecond branch of this preſent *Rule*, ſhall be the ſigne of that terme in the *quotient*, which ſtandeth directly vnder the laſt terme of the Diuidend; and then is the place of *primes* the next higher roome to that: wherefore the *prime* line is the next parallel (being alwaies that next aboue the place of *primes*, towardes the left hand) which let be marked out, and then are you ready for operation, your worke ſtanding thns. Whereby it now appeareth at

$$\begin{array}{r|l} 24\dot{6}\dot{3} & \dot{7}\dot{5} \\ 642 & \dot{3} \end{array}$$

firſt view, that the firſt terme in the *quotient* muſt needes bee *primes*, for that it is alwaies to be placed vnder the laſt of the Diuiſor, which here ſtandeth in the next lower roome, vnto the *prime* line, being alwaies the place of *primes*: wherefore now proceed in operation as formerly is taught for whole numbers, and you ſhall produce in *quotient* $38\dot{3}\dot{5}\dot{8}$ which is $\frac{9855}{25692}$ or in knowne parts of coine, 7 s-8 d $\frac{128}{2141}$ without vſe of fraction or reduction, as was required.

EXAM-

EXAMPLE III.

Two Inſtrumentall numbers being giuen, (the fractionall ſignes of the Diuidend being leſſe then thoſe of the Diuiſor) to diuide the one by the other without fraction or reduction.

LEt 8675$\frac{16}{17}$ and 138$\frac{25}{25}$, bee two inſtrumentall numbers giuen, whereof the firſt (being the Diuidend) is 8675$\frac{16}{17}$, or in knowne parts of coine, 8675 l- 12 s- 9$\frac{4}{17}$ d. and the other being the Diuiſor 13$\frac{25}{25}$, or in knowne parts of coine 13 l- 17 s- 3 d. the ſignes of the Diuidend being leſſe then thoſe of the Diuiſor: and let it bee required to diuide the one by the other, without fraction or reduction. Firſt place the giuen numbers in the Table, as before, thus: then conſidering that the ſignes of the Diuiſor, exceed thoſe of the Diuidend by 2, (the greateſt ſigne of the Diuidend being but 2, and that of the Diuiſor 4,) you are by the third branch of this preſent *Rule* to adde 2 places more vnto the Diuidend towardes the right hand, and to take the next following

8675	64
138	625

lowing line for the *prime* line: which being so taken and marked as before, the giuen numbers stand ready for operation thus. And now it appeareth at the first sight, that there will be in the *quotient*, 3 termes of Integers (besides the fractionall termes) for that there are so many places to the *prime* line, from the last of the Diuisor (accounting that for one) wherefore proceeding in operation, as before, you shall produce in your *quotient* $625\dot{8}\dot{3}\dot{5}\dot{1}$, or $625\frac{8351}{10000}$ or in knowne parts of coine, $625\,l\text{-}16\text{-}8\frac{488}{1209}$ which is somewhat lesse then $8\frac{1}{2}$ D. Being most speedily wrought without fraction or reduction, as was required.

$$\begin{array}{l} 86756\dot{4}\dot{0}\dot{0} \\ 138\dot{6}\dot{2}\dot{5}\cdot\cdot \end{array}$$

These few examples, with good respect and due obseruation, may well suffice for working all fractionall Diuisions whatsoeuer can bee giuen or required in whole numbers: Wherefore I will heere next shew and expresse certaine briefe and ready Rules in this operation, and so conclude the same.

RVLE

Rvle. VII.

To reduce ſhillings into pounds, or to diuide ſpeedily by 20.

LEt 356 bee a number of ſhillings giuen, and let it bee required to reduce the ſame into pounds, or to diuide it by 20. Firſt place the giuen number in the Table any where, ſtanding aboue the *prime* line thus: then remoue the giuen number a roome lower, or (which is all one) take the next higher line for the *prime* line, and then take halfe the giuen number for your *quotient*, and your work will ſtand thus: where you haue 17l-16s-0d. the number required.

	356
35	6
17	8

Rvle. VIII.

To reduce pence into ſhillings, or to diuide by 12.

LEt 167 bee a number of pence giuen, and let it bee required to diuide the ſame by 12, or to know how many ſhilling are therein. Firſt take ½ of the giuen,

uen, which is 27, and 5 remaining for ſo many pence, then take ⅓ of that 27, which is 12½ the 12 is ſo many ſhillings, and the ½ being 6 D. is to be added to the former 5, which together make a 11 S. So haue you 13 S. 11 D. The thing required.

RVLE. IX.

To reduce pence into pounds, or to diuide by 240.

LEt 3864 be a number giuen, and let it bee required to reduce the ſame into pounds, or to diuide it by 240. Firſt place the giuen number any where in the Table thus, (ſuppoſe the *prime* line that next following) then take the ſixth part thereof, asking how often 6 in 38, which is 6 times, and 2 remaining, wherefore cancell the 38, and place the 6 in the ſame roome where the 8 ſtood; then for the 2 remaining, and the next following 6, ſay how often 6 in 26, which is 4, and 2 remaining, wherefore cancell the laſt 6, and place 4 there, and for this 2 remaining, & the laſt terme 4, ſay how often 6 is 24, which is euenly 4 times, and nothing remaining; wherefore

3864

fore let that 4 ſtand and your firſt worke is finiſhed, ſo haue you now in ſtead of the giuen number, the ſixth part thereof, ſtanding thus in the Table. 644 Then are you either to remoue this number a roome lower, or which is all one thing) to leaue that former *prime* line, and take the next higher parallel for your *prime* line, and then ſhall your number ſtand thus: whereof you are now to take the $\frac{1}{4}$ by working, as before for $\frac{1}{2}$, ſo ſhall you inſtantly produce 16l–2s–0d. or 161 the number required, and thus ſhall your work then ſtand. 64|4 16|1

RVLE. X.

To diuide ſpeedily any number giuen by 5.

LEt 3648 be a number giuen, and let it be required to diuide the ſame by 5. Firſt place the giuen number in any place of the Table (ſuppoſing the next folowing line the *prime* line) ſtanding thus; then remoue the giuen number a roome lower, or rather at firſt making choice for the *prime* line of that 3648

next higher; & then shall your giuen number stand thus in the Table; wherefore now are you to double this number, where now it stands, and then it will thus appeare. So haue you 729½ or in knowne parts of coine 729 l–12 s–0 d. the *quotient* required.

364|8

729|6

RVLE XI.

To diuide any number giuen by 10. 100. 1000, *&c.*

LEt 36875 bee a number giuen, and let it bee required to diuide the same by any such Diuisor. First let the giuen number be any where placed in the Table thus: & if it be required to diuide the same by 10, take the next higher parallel for *prime* line, & then will it stand thus; which is 3687½ or in coine 3687 l–10 s. If you would diuide it by 100; then take the 2. higher parallel for *prime* line, and then will it stand thus inthe Table, which is 368¾ or in coine, 368 l–15 s. If by 1000, then take the third higher parallel for *prime* line, and then

36875|

3687|5

368|75

then will it ſtand thus; which is $36\frac{7}{8}$ or in coine 36l-17s-6d. 36|875

and if you would diuide the giuen number by 10000, take the fourth parallel for *prime* line (that is alwaies cutting off by the *prime* line, ſo many figures of the giuen number, as there are cyphers in the Diuiſor) and then will it ſtand thus; which is $3\frac{11}{16}$ or in coine, 3l-13s-9d. 3|6875

And the like of all others of the like kinde; which *Rule* holdeth generall, as well in Multiplication as Diuiſion, by only remouing the *prime* line higher or lower, without any operation at all, to multiply or diuide any number giuen, whereby the true value as well of product as *quotient*, appeareth at the firſt view.

Whereby it is apparant that if any number giuen, be to bee diuided by 30. 40. 600. 8000. *or the like, it is no more but remouing the Prime line ſo many places higher, as there are cyphers in the Diuiſor, and then to take the* $\frac{1}{3}$. $\frac{1}{4}$. $\frac{1}{8}$ *or* $\frac{1}{3}$ *of the whole number giuen, the* quotient *preſently appeareth.*

RVLE XII.

To diuide any number giuen by any aliquot *part of* 100. 1000, *&c.*

LEt 4627 be a number giuen, & let it be required to diuide the same by 25. First place the giuen number in any part of the Table thus. And considering that 25 is $\frac{1}{4}$ of a 100. take your *prime* line 2 roomes higher (that is as many roomes as there are cyphers in a 100.) and then will it appeare in the Table thus, wherefore now no more but multiply the whole number giuen by 4 (because 25 is $\frac{1}{4}$ of 100.) and your finished work stands thus; the product of that Multiplication being the quotient sought for, which is $185\frac{27}{100}$ or in coine 185 l. 1 s. $7\frac{1}{5}$ d.

	4627
46	27
46	27
185	08

Certaine properties of Diuision.

In all Diuisions the Diuidend is either equall to the Diuisor, or else is it greater or lesse then the same, if equall the *quotient* shall be alwaies an vnite, if greater

greater the *quotient* ſhall bee euer more then an vnite. If leſſe, then the *quotient* ſhall be a fraction leſſe then a vnite.

If the *quotient* found by any Diuiſor, doe againe diuide the firſt Diuidend, the ſecond *quotient* found ſhall be that which was the firſt Diuiſor: As if 24 be firſt diuided by 6, the *quotient* ſhall bee 4, and the ſame Diuidend being againe diuided by the ſame *quotient* 4, this ſecond *quotient* ſhall bee 6, which was the firſt Diuiſor.

If diuers particular numbers bee diuided by one number, and their ſeuerall *quotients* collected into one totall; and if alſo the ſame Diuiſor diuide the totall of the firſt particular numbers, this laſt *quotient* ſhall bee equall to the totall of the former *quotients*. As if 9. 12. 15. and 18. bee ſeuerally diuided by 3, their ſeuerall *quotients* ſhall bee 3. 4. 5. and 6, making together 18, then let the totall of thoſe particulars, 9. 12. 15. and 18, bee like diuided by the firſt Diuiſor 3. And I affirme that this laſt *quotient* ſhall bee equall to the totall of the former *quotients*, which is likewiſe 18.

The proofe of Multiplication and Diuiſion.

As formerly in Multiplication, the Multiplicand, Multiplier, Product and 1, make alwaies 4 proportionals; ſo heere alſo in this operation, the Diuidend, Diuiſor, *quotient* and 1, make euer 4 proportionals, and therefore are theſe two operations of Multiplication and Diuiſion, called proportionall operations, who yet in power & nature are abſolute contraries; whereby it commeth to paſſe that each of theſe operations are the others triall. As firſt for Multiplication, if the product bee diuided by the Multiplier, the Multiplicand is produced in *quotient*, if by the Multiplicand, the Multiplier is produced. And then for Diuiſion; If the *quotient* bee multiplied by the Diuiſor, and the remainder (if any be) added to the product, the totall ſhall be the Diuidend, if truely wrought.

Of Geometricall proportionality discontinuall, and of the GOLDEN RVLE *depending thereon.*

FOr as much as I bend mine indeuours in this small volume to be as briefe as possible I may, I wil here forbeare to handle at large the seuerall sorts & kindes of proportionalities, with the seuerall properties, effects, and operations thereof; and considering also that the principall Rules, conclusions, and most curious secrets of Arithmeticke, depend on this *Geometricall proportionality discontinuall*, I will content my selfe (with as much breuity as may bee) to giue a taste onely thereof, and to shew by some few Examples, the practike operation therein, by my *Arithmeticall Iewell*; whereby, and with due respect and obseruation had of the precedent Rules, the ingenious Practitioner may easily attaine to the knowledge and most speedy operation

ration of many excellent conclusions Arithmeticall.

Definition.

Proportion disioyned or discontinuall, is when any foure termes are so taken and placed, that the first two haue the same relation either to other, as the last two haue.

AS for example, let these 4 termes be propounded, 4. 2. 6. 3, which are thus to be conferred Geometrically, as 4 is to 2, so is 6 to 3, wherein it is euident that the comparison or relation of the two first termes, is equall and all one with that of the two latter termes; for either Antecedent is in double proportion to his proper consequent, as 4 is twice 2; so is 6 twice 3: but the two meanes, or middle termes are in proportion, *subtripla*, 2 being in 6, thrise or 3 times: and therefore is this proportion called disioined or discontinuall, becausse the third terme hath not the same proportion to the second, as the second to the first.

This proportionality discontinuall, consisteth of two kindes; Direct, and Reciprocall; which is vsually termed conuersed,

Direct

Direct, is when the termes are conferred directly forwards, from the first to the second, and from the third to the fourth: and conuerſed or reciprocall is when they are diſlocally conferred, that is, from the first to the third, and then backewardly, from the fourth to the ſecond.

Concerning proportionalty Direct, let theſe 4 termes bee giuen, 12. 6. 8. 4. Now I ſay theſe 4 termes, as here they ſtand directly, or rather being placed thus; $\frac{12}{6}$ and $\frac{8}{4}$ (as proportionals ought to be placed) are called proportionals directly; becauſe the firſt couple are compared together by themſelues, each to his owne fellow, and likewiſe the latter; both antecedents being compared to their proper conſequents, for as 12 is to 6, ſo is 8 to 4, proceeding forward directly.

But let thoſe termes, on any occaſion in worke be giuen & placed thus; 12. 4. 6. 8. or thus, $\frac{12}{4}$ and $\frac{6}{8}$ heere, if they were now compared directly, as heere they ſtand, they were altogether diſproportionall; but compare the firſt to the third, and the fourth to the ſecond, that is, 12. to 6, & 8 to 4, the compariſons are equall, and the proportionalty perfect: & therefore I ſay,

I say, by reason of this backeward comparison, is this properly called proportionalty *conuersed* or *reciprocall*.

The maine effect of this proportionality discontinuall is, by 3 numbers giuen or knowne, to finde out a fourth proportionall number; wherefore in respect of the operation, this is commonly called *The Rule of three*; but in regard of the excellency thereof, it is as vsually termed *The Golden Rule*: the reason and ground of which *Rule* dependeth on this *Theoreme*.

If foure numbers be proportionall, the product made by the Multiplication of the 2 meanes, shall be equall to the product of the two extreames. Euc. 7. 19.

As in the former example, where 12. 6. 8. & 4. are 4 proportional terms giuen, whereof 12 and 4 are the two extreames, and 6 and 8 are the two meanes: Now if 6 and 8, the two meanes multiply the one the other, their product shall be 48; and the same likewise is the product of 12 and 4, the two extreames, being multiplied the one by the other, as this *prop.* importeth. Wherefore if any three of those termes are giuen, the fourth is readily

readily found ; as ſuppoſing the greater extreame 12, and the two meanes 6 and 8 were giuen, and the leſſe extreame 4 were ſought for : reaſon thus with this *Rule*, If 12 giue 6, what giues 8, or if 12 giue 8, what giues 6 ; and multiplying 6 by 8, or 8 by 6, which is all one thing, & diuiding the product by 12, the *quotient* ſhall be 4, the leſſe extreame ſought for; and the like of the reſt.

Now when by this Rule, ſuch a fourth proportionall number is found (if the foure numbers be contract to particular Denominations, or kindes, as vſually they are) then are two of them of one kinde and Denomination, and the other two of another, which couples ſo agreeing are called, Match termes : the one couple being the antecedents, and the other the conſequents.

Of which foure terms, if the third being greater then the firſt (which with it matcheth) doth therefore (as in reaſon it ought) require a greater portion, or being leſſe, doth therefore require a leſſe; the are both ſuch proprtionalities direct, & are called, *The Golden Rule*, or *Rule of three direct*. As thus, let this queſtion bee demanded; If 4 moneths diet coſt 8

pound

pound, what coſt 6 moneths diet.

Here are three numbers, or termes giuen, 4. 8. and 6, whoſe fourth proportionall by operation wil appeare to bee 12; for as 4 the firſt terme is halfe of 8, being the ſecond terme, ſo is 6 the third terme, halfe of 12 the fourth terme.

Againe, as the firſt and third termes, 4 and 6, haue the particular Denomination of months, ſo the ſecond and fourth termes 8 and 12, haue the particular Denominations of pounds in mony, wherefore theſe termes of like Denomination, are called Match termes.

Likewiſe in reſpect that 6 months, the third terme being greater then 4, his Match terme, doth by reaſon thereof require a greater price; or number of pounds then, 4 months his match terme did; And in regard (if the queſtion were ſo put) that if 6 months diet coſt 12 l. then 4 (being a leſſe number of months) muſt of neceſſitie coſt leſſe, ſeeing that in either queſtion, more, requires more, and leſſe requireth leſſe. I therefore conclude, that both thoſe proportionalties are direct.

But now ſuppoſe that the third terme, being greater, or exceeding his Match

terme; doth notwithſtanding craue ſo much the leſſe for his portion, or being leſſe, requireth ſo much the more, then is ſuch proportionalty called the *Backeward Rule of three*, or the *Golden Rule conuerſed*; becauſe here, as the firſt terme is to the third directly, ſo is the fourth to the ſecond conuerſedly, as thus:

If 12 buſhels of Oates ſerue 8 Geldings 12 dayes, how many will the ſame finde 16 Geldings.

Now conſidering that 16 Geldings, being the third terme, is greater or more then 8 Geldings his Match-term, and yet in reaſon requireth leſſe time, or fewer dayes then 8 Geldings (for by operation and common reaſon, the ſame quantity which ſerueth 8 Geldings 12 daies, will ſerue 16 Geldings but 6 daies) the conference of 8, the firſt terme to 16 the third terme directly forward, is the ſame of 6, the fourth terme to 12, the ſecond backewardly, the termes ſtanding thus; 8. 12. 16. 6: where the firſt antecedent, is compared to the ſecond directly; and the laſt conſequent to the firſt, conuerſedly: wherefore I conclude this proportionallity is to bee wrought by the Rule of three conuerſed. And the like,

if

if the question were so changed that the lesse should require more.

And thus having briefly touched the propertie, passion, and nature of this proportionallity discontinuall, & the Golden Rule. I will now proceed in shewing the order, forme, and ordinary practise of this Rule, both direct, and conuerted, with my Table, by some few examples to that purpose.

RVLE I.

Three whole numbers being giuen to finde a fourth proportionall, in direct proportion discontinuall.

LEt 10. 12. and 348, bee three whole numbers giuen, and let it be required to finde a fourth proportionall number, in direct proportion discontinuall. Or let this question bee propounded: If a staffe pitched in the earth, perpendicularly 12 foot aboue the ground, cast a shadow 10 foot in length, what height is that steeple of whose instant shaddow is 348 foot long. First, considering that in all works of this nature, either by the disorderly propounding of questions, or otherwise, the

the first, second, or third number in the question, is not alwaies the first, second, or third number in place of proportion, (as here 12, the first number of the present question, is the second number or first consequent in place of proportion) whereby oftentimes the weake practioner is more pestred in placing his termes, then working the operation. Let this bee therefore an infallible rule for your satisfaction heerein: Consider alwaies in any question, on which of the 3 giuen numbers the question dependeth, and take that alwayes for the third number, or latter antecedent (as well in this rule Direct, as in the backward rule) as here 348 is the number whereon the question dependeth, thus; if 20 foot shadow come of 12 foot height, or giue 12 foot height, what 348 the third number? which hath alwaies in al questions for his badge or cognisance, an Interogation before him, as what? how many? how much? or the like, and being thus found in all operations by the rule of three Direct, it is euer most conueniently placed, in naturall order towards the right hand, in the first and highest ranke of the Table; then the first terme in proportion, or first antecedent

tecedent is alwayes that which is of the ſame Denomination or Match-terme with the third, which in this example is 20, both their Denominators being ſhadow; which firſt terme (being the Diuiſor) is euer moſt fitly placed in the third ranke of the Table, directly vnder the former, and then of neceſſity the terme remaining, which in this example is 12, muſt bee the ſecond terme in proportion, or firſt conſequent, being Match-terme to the fourth number ſought for, and ought euer to bee placed in the 2 ranke betweene the two former numbers, but moſt conſequently for operation, as is preſcribed in *Rule* 3, of *Operation* 3, and as here it is placed. So ſhall the numbers in your Table ſtand thus; moſt apt & ready for preſent operation (and no way ſo conueniently.)

348	
12	
20	

Wherefore now worke thus. Firſt by *Rule* 3, or 7 of *Operation* 3, multiply 348 by 12, & the product ſhall be 4176, then cancelling your multiplier 12 (as hauing no further vſe thereof) and remoue your Diuiſor 20, into the ſame ſecond ranke, and then according to *Rule* 1 of *Opera:* 4, diuide the product 4176, by your Diuiſor 20, or rather ſeeing your

your Diuiſor is 20, worke by *Rule* 7, of *Oper.* 4. So ſhall you ſpeedily produce in *quotient* 208⅝ your finiſhed worke ſtanding in the Table thus. So haue you ſpeedily found a fourth proportionall, to the three giuen termes, and that if the ſhadow of 20 foot come of any perpendicular height of 12, that then any ſhadow of 348 foot, comes from a perpendicular height 208⅝ the thing required.

417|6
208⅝

In all operations by the Rule of 3 direct place euer (as before) the firſt proportionall terme in the third ranke of the Table, the ſecond in the ſecond, & the third in the firſt. So ſtand your numbers ready for preſent operation, without tranſmutation of numbers, prepoſterous working, or want of roome.

A ſecond Example of the former Rule.

LEt 156. 100 and 540, be three whole numbers giuen, and let it be required to finde a fourth proportionall number, in direct proportion diſcontinuall. Or let vs make bold to borrow this queſtion of a friend (who hath wrought the ſame by

by his Rule of *false position*) A man dying bequeathed a legacy with this prouision, that as long as the same remained in his Executors hands, hee should allow vnto the Legator for the same, after the rate of 8 *per cent.* simple Interest: At 7 yeeres end, the Executor paid vnto this Legator, in full satisfaction of the legacy and interest (after the former rate) 540 l. The question is what was the first legacy giuen. In all questions of this nature, it is no more but first to consider what is the interest of a 100 l. after the giuen rate for the terme limitted, as here 8 *per cent.* for 7 yeeres appeareth at first view to bee 56 l. for 7 times 8 is 56, wherefore now ioyning with it a 100 l. the principall, your three proportionall termes are apparant, and your question lies thus. If 156 l. principall and Interest, come of a 100 l. principall, of what principall comes 540 l. which by this present Rule of 3 Direct (*without the Rule of false position*) is presently answered thus. First place your termes in the Table (as before is taught) where they will thus appeare. Then according to the Glosse of *Rule* 6, *Operation* 3, multiply the

540
100
156

the first number by the second, as heere they stand, that is, 540. by 100. the product shall be 54000, which by *Rule* 1. 4. being diuided by 156, the *quotient* shall be $346\frac{3}{13}$ the fourth proportionall sought for, or in answere of the question propounded 346 l–3 s–$0\frac{3}{4}$ d. and $\frac{3}{13}$ of $\frac{1}{3}$ the thing required.

Rvle II.

Three fractionall termes or instrumentall numbers being giuen, to finde a fourth proportionall indirect proportion discontinuall; without fraction, addition, or reduction.

LEt $24\frac{3}{4}$ $19\frac{13}{16}$ and $324\frac{1}{2}$ bee 3 fractionall termes giuen, whereof the first importeth in instrumentall numbers, 247, or in coine, 241–15 s. the second 198, 25 or in coine 19 l–16–3 d. and the third, 3245 or in coine 324 l–10 s. and let it be required without vse of fraction addition, or reduction, to finde a fourth proportionall in direct proportion discontinuall. Or let this question bee propounded, If $24\frac{3}{4}$ yards of satin cost 19 l–16–3 d. what cost $324\frac{1}{2}$ yards at that rate In all questions

questions of this nature place your instrumental termes in the Table, as before is taught; where thus they will appeare, by *Rule* 4. 3. multiply the first by the second, as now they are placed, and the product shall be then cancell the multiplier, and place in due order (as before is taught) your Diuisor 24$\dot{7}\dot{5}$ in the same second ranke where formerly your multiplier stood; and then thereby according to *Rule* 6. 4. *Example* 2, diuide your former product, and your *quotient* shall produce 259$\dot{7}\dot{6}\dot{3}\dot{8}\dot{8}$. 259$\frac{605}{792}$ the fourth proportionall sought for; or in answer to the question propounded 259 l-15-s 3$\frac{1}{3}$ d. being most speedily wrought, without fraction, addition, or reduction, as was required.

$$\begin{array}{r} 3245 \\ 198\dot{1}\dot{2}\dot{5} \\ 24\dot{7}\dot{5} \\ \hline 6429\dot{1}\dot{5}\dot{6}\dot{2}\dot{5} \end{array}$$

It is alwaies to bee remembred in these workes, that assoone as your termes are placed in the Table (before you beginne your multiplication) you are to marke out your Prime line (as before is taught) and hauing thereby receiued the value of the product, and placed your Diuisor as before, ready for Diuision; then are you to alter your former prime line, and to

marke

marke out that which must serue for your Diuision, as before is likewise taught; so shall this serue for discouery of your quotient, as that of your product.

Of proportionallity conuerſed, or the backeward Rule of three.

IT is first to bee vnderstood that in this rule, the proportionall termes are to keepe the selfe same order as they did in the rule of three Direct; so that as well heere as there, that terme is euer to be reputed the third number in proportion, whereon the question dependeth, and the rest in the same order as before: But herein consisteth the difference in operation, that where alwaies in the rule of 3 direct, you were to multiply the second terme by the third, and to diuide the product by the first, in this backeward rule you are euer to multiply the first terme, by the second, and to diuide the product by the third; and therefore it is most conuenient in all operations by this rule, to place the seuerall termes in your Table as they stand in order of proportion; that is, the first terme in the first ranke, the second in the second, and the third in the third; so shall they stand most apt and

ready for all operation by this rule, which being vnderstood wee will proceed to practise therein.

Rvle III.

Three whole numbers being giuen, to finde a fourth proportionall in conuersed proportion discontinuall.

LEt 350. 12 and 135, be three whole numbers giuen, and let it be required to finde a fourth proportionall in conuersed proportion discontinuall. Or let this question be propounded, *A.* lent vnto *B.* 350l. for 12 months, with condition that *B.* should doe the like for *A.* when he had cause of vse, who being afterward required thereunto, could spare vnto *A.* but 135 l. wherefore the question is, how long *B.* shall forbeare this money, to make vnto *A.* a iust recompence for his former kindnes. The question is plainly framed thus. If 350l. were forborne 12 months, how long must 135l. be forborne wherein is to bee noted, that the third number 135, being lesse then the first his match-terme 350. doth yet require more, that is more time of forbearance, (as by reason it ought) whereby it is apparant that this proportionalitie is conuersed; and therefore thus to bee wrought: First place your

termes

termes in the Table orderly, as they ſtand in proportion, that is the firſt in the firſt ranke, the ſecond in the ſecond, & the third in the third, where thus they will appeare.

350	
12	
135	

And now by *Rule* 3 or 7, of *Operation* 3. multiply 350 by 12, the firſt by the ſecond, and the product ſhal be 4200, then cancell your multiplier 12, and remoue your diuiſor 135 into the ſame ſecond ſecond ranke; whereby according to *Rule* 14. diuide the former product, and the *quotient* ſhall bee $31\frac{3}{9}$ the fourth proportionall ſought for, as was required.

RVLE. IIII.

Three fractionall termes, or inſtrumentall numbers being giuen, to finde a fourth proportionall, in conuerſed proportion diſcontinuall, without vſing fraction, addition, or reduction.

LEt $1\frac{1}{3}$. $54\frac{1}{3}$ and $2\frac{11}{4}$ bee three fractionall termes giuen, whereof the firſt importeth in inſtrumentall numbers 12, or in coine 24 s. the ſecond 544, or in coine 54 l-8 s. and the third 2175, or in coine 21 l-15 s-6 d. and let it be required without vſe of fraction, reduction, or addition, to finde a fourth proportionall in

conuersed proportion discontinuall. Or let this question be propounded: If when a quarter of wheat be sold for 24 £. the two penny white loafe (by the statute) ought to weigh $54\frac{2}{5}$ ounces, what ought the same loafe to weigh when the quarter of wheat is sold for 45 £–6 D. First place the termes in the Table directly (as in the last rule) where they will stand thus: then by Rule 4. 3. multiply the first by the second, and the product shall bee $6\dot{5}\dot{2}\dot{8}$, then cancell your multiplier and remoue the diuisor $2\dot{2}\dot{7}\dot{5}$ into the same second ranke where your multiplier stood; and by *Rule* 6. 4. *Example* 2, diuide the same product by your diuisor $2\dot{2}\dot{7}\dot{5}$ and the *quotient* shall bee $28\dot{0}\dot{9}\dot{4}\dot{5}$ or $28\frac{316}{455}$ the fourth proportionall required.

$$\begin{array}{r} 1\dot{2} \\ 54\dot{4} \\ 2\dot{2}\dot{7}\dot{5} \end{array}$$

HEere I might now proceed aswell in shewing variety of Examples, in the seuerall kindes & rules of proportions plural, and compounded of Society, Allegation, False Positions, Practise, & the like: As with what incredible ease, perspicuity and speed not only these, but all other Arithmeticall operations are, and may bee performed by this little Table. But intending not hereby to erect a free Schoole of Arithmetike, or to performe what can be; but rather briefely giuing taste what may be effected: I heere abruptly cease, leauing this little Worke (in great loue) to thy good liking, and all of vs to Gods protection.

FINIS.